Keynesian Macrodynamics: Convergence, Roads to Instability and the Emergence of Complex Business Fluctuations

Carl Chiarella[*], Hing Hung[*], Peter Flaschel[**]

Abstract We reformulate the traditional AS-AD growth model, with a Taylor policy rule replacing the conventional LM-curve. The essential features of the model are gradually adjusting wages and prices, perfect foresight on current inflation rates and an adaptive revision of the inflationary climate in which the economy is operating. We compare this approach with the New Keynesian approach with staggered price and wage setting and find that whilst both approaches have common components, they have radically different dynamic implications due to the treatment of the forward-looking part of our wage-price spiral. We show that an estimated version of our model implies local asymptotic stability, due to stable interaction of goods market dynamics with the interest rate policy rule of the central bank, and due to a normal working of a real-wage feedback chain. These results are however endangered when there is a global floor to money wage inflation rates, leading in fact to economic breakdown. In this latter case, the return of some money wage flexibility in deep depressions is of help in restoring the viability of the model, thereby avoiding explosive dynamics and the collapse of the economy.

Keywords Keynesian dynamics, wage and price Phillips curves, persistent business cycles, complex dynamics

JEL classification E24, E31, E32

1. Introduction

In this paper we present the analysis of an empirically motivated model of D(isequilibrium)AS-D(isequilibrium)AD type. Its origins as far as the considered wage-price spiral is concerned date back to the paper of Chiarella and Flaschel (1996). The wage-price mechanism has been extended and studied in detail, from the analytical as well as from the quantitative perspective, in Chiarella et al. (2005). The results obtained in Chiarella et al. (2005) and which we extend in the present paper for the postulated wage-price dynamics, augmented by a (partly) conventional Keynesian goods market dynamics, Okun's law and a conventional type of a Taylor interest rate policy rule stand in striking contrast—despite formal similarities—to the ones obtained from the com-

---

[*] School of Finance and Economics, University of Technology Sydney, P.O. Box 123, Broadway NSW 2007, Australia.

[**] Corresponding author. Bielefeld University, Department of Business Administration and Economics, Universitätsstraße 25, 33615 Bielefeld, Germany. E-mail: pflaschel@wiwi.uni-bielefeld.de, Phone: +49521123075.

parable New Keynesian macrodynamics which assumes a staggered wage and price setting. We use estimated parameter sets to study the stability features of our model numerically as well as analytically. The convergence properties that we obtain in this way are however, by and large, due to the linear version of the estimated DAS-DAD system, since the employed time series are bounded and therefore cannot generate and observe locally explosive behavior which becomes bounded through the establishment of behavioral changes when the system departs too much from the balanced growth path of the model.

These convergence properties will disappear (in the downward direction) when this model type is enhanced by downward money wage inflexibility, as observed by Hoogenveen and Kuipers (2000) for six European countries, since our estimated DAS-DAD model is profit-led (where real wages increases are contractionary due to a dominance of investment over consumption). In a profit-led economy with downwardly rigid wages and (maybe only sluggishly) falling prices, it turns out that in a depressed situation real wages must increase, a fact which makes the ongoing depression ever deeper, until the economy either collapses or undergoes a significant change in behavior.

We also consider the case in which the economy is wage-led (where real wages increases are expansionary due to a dominance of consumption over investment), where therefore increasing wage flexibility is bad for economic stability, since booms will tend to produce real wage increases which further stimulate the economy under such a regime. The same accelerator mechanism is of course then also working in a downward direction, but can obviously (from a partial perspective) be stopped if there are floors to money wage deflation. In a wage-led regime a kink in the money wage Phillips curve therefore can limit the purely explosive behavior that exists in the unrestricted case and can indeed be shown to lead—even in a fairly advanced five dimensional system—to bounded and in fact economically viable dynamics.

These dynamics moreover will be of a complex type if the unrestricted dynamics becomes explosive enough around the steady state. Our model is thus able to generate, quite naturally and without the imposition of economically unrealistic parameter values, the types of complex economic behavior that has been written about (among others Rosser 1999, 2000).

We thus find interesting dynamical features in an advanced Keynesian aggregate supply/aggregate demand model with sluggish price, wage and output adjustments that allow for convergence results (in particular for an estimated version of the model) and thus for considerations that relate to the Frisch paradigm in business cycle theory. However, not unrelated to the estimated sizes of parameter values, we can also find situations where endogenously generated irregular business fluctuations are observed, which in our view confirm (though with time-invariant parameters still), the cycle theory advanced in Keynes' (1936) General Theory, which we would characterize as an anti-Frisch or more aptly as a Keynesian paradigm.

The paper develops as follows. In Section 2 we introduce the basic relationships of our Disequilibrium Aggregate Supply – Disequilibrium Aggregate Demand model and derive its reduced form dynamics. In Section 3 we provide an estimated version of the model, obtained in earlier work. We then simulate the model to gauge its response to

positive real wage shocks. We also carry out an eigenvalue analysis around the steady state of the model in order to determine which parameters are most likely to be destabilizing. Section 4 discusses the stability properties of the model and proves a number of propositions about the stabilizing/destabilizing tendencies of various parameters. In Section 5 we introduce a mid-range downward wage rigidity and discuss its role in stabilizing the model when it is subject to explosive fluctuations. Section 6 gives further simulations of the model with parameters chosen close to those of the estimated model, but now allowing a situation in which wage flexibility with respect to demand pressure is destabilizing. By analyzing phase plane projections and bifurcation diagrams we show how the dynamics can easily become complex. Section 7 draws some conclusions.

2. Baseline DAS-DAD macrodynamics

In this section we reconsider a model of the D(isequilibrium)AS-D(isequilibrium)AD variety, introduced in Chen et al. (2006) as an empirically motivated reformulation of a baseline model of the Keynesian D(isequilibrium)AS-AD variety investigated in Asada et al. (2006). These model types can be characterized as matured redesigns of the standard model of the old Neoclassical Synthesis and will be contrasted with the corresponding model type of the now fashionable New (Keynesian) Neoclassical Synthesis. In our baseline models (with their dynamic or static formulations of goods market behavior) we avoid the logical inconsistencies of the old Neoclassical Synthesis, described in detail in Asada et al. (2006). Basically this is achieved by formulating a wage-price spiral mechanism consisting of a money wage Phillips curve (WPC) and a price Phillips curve (PPC) that at first sight look very similar (from the formal perspective) to the wage-price dynamics of the New Keynesian model, when both staggered prices and wages are considered in the latter approach.

In our first formulation of the wage-price spiral in Chiarella and Flaschel (1996), we saw qualitatively the same variables and the same parameter signs on the right-hand sides of the two Phillips curves (PC's) as far as the dependence of wage and price inflation on output and wage gaps are concerned. But in that earlier work on Keynesian macrodynamics, hybrid inflationary expectations formation for the accelerator terms were always used. Here in our formulation of wage and price inflationary expectations formation, we have on the one hand myopic perfect foresight, not as in the case of the New Keynesian wage-price dynamics on the own one-period-ahead rate of inflation (a self-reference mechanism), but rather with respect to other rates of inflation (a hetero-reference mechanism), as seems appropriate when one speaks of cost-pressure items in the tradition of mainstream Phillips curve formulations. Due to this cross-over structure in the myopic perfect foresight component of the PC accelerator terms we can furthermore assume a neoclassical dating of these expectations, meaning thereby that time indices of inflation rates are the same on both sides of these two PC's, whereas the New Keynesian Phillips Curves use the current rate and the one-period-ahead wage inflation (or price inflation) rate on the left- and right-hand sides of their staggered wage (respectively price) adjustment rules. In addition, in our formulation we always include

backward looking expectations, but interpret such (adaptively updated) expectations as an inflationary climate expression, which agents form in addition to their correct myopic expectations in order to also take into consideration the inflationary regime into which current inflation is embedded.

In comparison to the New Keynesian wage-price dynamics, see Woodford (2003) for example,[1] that may be written as

$$d\ln w_t = E_t(d\ln w_{t+1}) + \beta_{wy}y_t - \beta_{w\omega}\ln\omega_t, \tag{1}$$
$$d\ln p_t = E_t(d\ln p_{t+1}) + \beta_{py}y_t + \beta_{p\omega}\ln\omega_t.$$

We instead make use of the following representation of a Keynesian wage-price spiral (however still using the New Keynesian measures for the output and the wage gap):

$$d\ln w_{t+1} \stackrel{WPC}{=} \kappa_w E_t(d\ln p_{t+1}) + (1-\kappa_w)\pi_t^c + \beta_{wy}\ln Y_t - \beta_{w\omega}\ln\omega_t, \tag{2}$$
$$d\ln p_{t+1} \stackrel{PPC}{=} \kappa_p E_t(d\ln w_{t+1}) + (1-\kappa_p)\pi_t^c + \beta_{py}\ln Y_t + \beta_{p\omega}\ln\omega_t,$$

where w denotes nominal wage, ω real wage, and p price. The κ_w, κ_p, β_{wy}, $\beta_{w\omega}$, β_{py}, $\beta_{p\omega}$ are parameters and π^c denotes an inflationary climate variable, here updated by a standard adaptive expectations process in order to simplify the analysis of the model. Apart from the expectations expressions we thus have the same formal structure in these wage-price dynamics as in the New Keynesian formulation. Expectations in our formulation are now based on weighted averages of corresponding cost pressure terms, combining myopic perfect foresight with sluggishly adjusting inflation regime expectations. Besides the significantly different way of treating forward- and backward-looking expectations just discussed, the difference between the New Keynesian and our approach lies in different microfoundations of the wage and price PC's, based on what has been shown in Blanchard and Katz (1999). Reformulated in a deterministic continuous-time framework, the New Keynesian wage-price dynamics (1) read

$$\dot{\pi}^w = -\beta_{wy}y + \beta_{w\omega}\theta, \quad \dot{\pi}^p = -\beta_{py}y - \beta_{p\omega}\theta, \tag{3}$$

where π^w and π^p denote the wage and price inflation rates, respectively, and $\theta = \ln\omega$. Reformulating our approach (2) in continuous time—now specifically in terms of the employment rate e on the labor market and the capacity utilization rate u on the market for goods (so that the coefficients β_{wy}, β_{py} reappear as β_{we}, β_{pu}) then gives rise to:[2]

$$\hat{w} = \pi^w = \beta_{we}(e-1) - \beta_{w\omega}\ln\omega + \kappa_w\hat{p} + (1-\kappa_w)\pi^c, \quad \kappa_w \in (0,1), \tag{4}$$

$$\hat{p} = \pi^p = \beta_{pu}(u-1) + \beta_{p\omega}\ln\omega + \kappa_p\hat{w} + (1-\kappa_p)\pi^c, \quad \kappa_p \in (0,1). \tag{5}$$

One difference between our approach and that of the New Keynesians to wage price dynamics therefore is that we use on the left hand sides inflation rates $\hat{w}$, $\hat{p}$ for wage and

---

[1] We set $y_t = \ln Y_t$ the log of current output and d the backward difference operator. Note also that we ignore in these equations the discount parameter $\beta \leq 1$ ($\beta \approx 1$) of the New Keynesian approach by setting it equal to one for expositional simplicity.

[2] We set $\hat{x} = \dot{x}/x$ the growth rate of a variable x. Note also, that—as in the New Keynesian approach—we have normalized steady state rates to 1.

price inflation in place of their time rate of change. Another important difference is that we use two measures for the output gap, one relating to the labor market (the deviation of the rate of employment from the NAIRU rate of employment $\bar{e} = 1$) and one relating to the goods market (the deviation of the rate of capacity utilization of firms from their intended normal rate of capacity utilization $\bar{u} = 1$). Furthermore in both the WPC and the PPC there appear weighted averages for the cost-pressure measures of both workers and firms, based on myopic perfect foresight and on our concept of an inflation climate variable π^c. Due to these difference in wage and price inflation formation, we do not get the sign reversal in front of the output and wage gaps that is typical for the New Keynesian approach to wage and price dynamics.

The WPC has been microfounded in Blanchard and Katz (1999) from the perspective of current theories of the labor market. Note that the log that appears in the formal representation of our WPC is not due to a loglinear approximation of the originally given structural equation, but instead results (see Blanchard and Katz 1999) from a growth rate reformulation of a bargained real-wage wage curve initially represented in level form. For the PPC a formally similar procedure is available, based on an elaborate approach to flexible markup pricing. The two PC's can be considered as a linear system of equations in the variable $\hat{w} - \pi^c$, $\hat{p} - \pi^c$ that can be easily solved, giving rise thereby to the law of motion (9) for the real wage $\omega = w/p$ and the reduced form price PC (11). Together with the law of motion for the inflationary climate expression (10) we therefore obtain all together three laws of motion that describe the Disequilibrium adjustment of Aggregate Supply of our model (the DAS-component) as is shown below.

The Disequilibrium AD part of the model (the DAD component) is here still of a simple type, consisting of a dynamic multiplier equation (6) in terms of the rate of capacity utilization u, whose rate of growth is assumed to depend negatively on its level u (as in the simple textbook multiplier story), as usual on the (here actual) real rate of interest $r - \hat{p}$ and (with ambiguous sign) on the real wage, which measures the impact of income distribution on the determination of aggregate demand and resulting output adjustments. We call a regime where $-\alpha_{u\omega}$ applies *a profit-led regime* and the opposite case a *wage-led regime*, characterizing in this way the situations where investment dominates consumption with respect to real wage changes and vice versa.

In addition to the law of motion for the rate of capacity utilization, we assume that the rate of employment e is obeying some sort of Okun's law, following the rate of capacity utilization with a time delay as shown in equation (7). We finally have a standard form of a Taylor interest rate policy rule (8), including however interest rate smoothing, in order to close the model. Note that parameter values are indexed in a way similar to the notation used in input-output tables and that all equations have been expressed in linearized form around their steady state values, which are here supplied from the outside and thus exogenously given. Taken together the *dynamic DAS-DAD macromodel*, with real wage dynamics now in the place of the nominal WPC and PPC, thus consists of the following five laws of motion (note that all parameters are positive),

$$\hat{u} = -\alpha_{uu}(u-1) - \alpha_{ur}((r-\hat{p}) - (r_o - \bar{\pi})) \pm \alpha_{u\omega}\ln\omega, \tag{6}$$

$$\hat{e} = \beta_{eu}(u-1) + \beta_{e\hat{u}}\hat{u}, \tag{7}$$

$$\dot{r} = -\gamma_{rr}(r - r_o) + \gamma_{rp}(\hat{p} - \bar{\pi}) + \gamma_{ru}(u - 1), \tag{8}$$

$$\hat{\omega} = \kappa[(1 - \kappa_p)(\beta_{we}(e - 1) - \beta_{w\omega} \ln \omega) - (1 - \kappa_w)(\beta_{pu}(u - 1) + \beta_{p\omega} \ln \omega)], \tag{9}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c), \tag{10}$$

which represent respectively the IS-dynamics, Okun's Law, the Taylor Rule, the dynamics of income distribution or of the real wage, and the updating of the inflationary climate expression. Since steady state values are here parameters of the model we assume for reasons of numerical simplicity that they are given by 1 in the case of utilization rates and real wages and—from an annualized perspective—by 0.1 and 0.02 as far as the steady state rate of interest and the inflation target of the central bank are concerned.

We have to use in addition the following reduced form expression for the price inflation PC,[3]

$$\hat{p} = \kappa[\beta_{pu}(u - 1) + \beta_{p\omega} \ln \omega + \kappa_p(\beta_{we}(e - 1) - \beta_{w\omega} \ln \omega)] + \pi^c, \tag{11}$$

which has to be inserted into the above laws of motion in various places in order to obtain an autonomous system of differential equations in the state variables: capacity utilization u, the rate of employment e, the nominal rate of interest r, the real wage rate ω, and the inflationary climate expression π^c. We have written the laws of motion in an order that first presents the dynamic equations also present in the baseline New Keynesian model of inflation dynamics, and then the extension with our dynamics of income distribution and the inflationary climate in which the economy is operating. This modification and extension of the baseline DAS-AD model of Asada et al. (2006) goes beyond this earlier approach to the extent that it now also allows for positive effects of real wage changes on aggregate demand (so-called wage led regimes), not yet present in the AD component of our original modification of the conventional AS-AD dynamics (which was always profit-led, due to the lack of an effect of income distribution on households' consumption).

When later comparing this model of DAS-DAD dynamics with its New Keynesian counterpart we will find from the theoretical and empirical perspective that the laws of motion of the DAS-DAD model imply damped oscillations if the inflation climate is adjusting sufficiently sluggishly and if price inflation rates respond sluggishly to the excess demand on the market for goods, while the New Keynesian version is no longer determinate in the case of staggered wage and price adjustment even for active monetary policy rules (as was the case for its baseline version). The New Keynesian variant therefore needs at the least more advanced Taylor interest rate rules than the conventional one, in order to get determinacy and thus unique responses to disturbances of its deterministic core dynamics. The DAS-DAD dynamics, by contrast, can become explosive if inflationary climate expectations (or even price inflation itself) are adjusting too fast. The resulting unboundedness must then be tamed by assuming behavioral nonlinearities at least far off the steady state that then limit the explosive

---

[3] The reduced form expression for the wage inflation PC reads: $\hat{w} = \kappa[\beta_{we}(e - 1) - \beta_{w\omega} \ln \omega + \kappa_w(\beta_{pu}(u - 1) + \beta_{p\omega} \ln \omega)] + \pi^c$.

nature of the dynamics such that they become bounded and thus economically viable. Strategies for finding meaningful trajectories for the New Keynesian and our matured Keynesian macrodynamics therefore differ radically from each other. They also have different implications with regard to the underlying paradigms of Frisch and Keynes, characterized by persistently shocked shock absorbers on the one hand and endogenously created business fluctuations (locally explosive dynamics – under certain side conditions on adjustment speeds—tamed by behavioral nonlinearities far off the steady state) on the other hand.

3. Simulating an estimated version of the model

The model (6)–(10) has been estimated for the U.S. economy (1965/1–2000/3) in Chen et al. (2006) with the following result for parameter values, obtained from a two-stage least-squares system estimate where the inflationary climate was measured by a twelve quarter moving average with linearly declining weights denoted by π_t^{12}:

$$\begin{aligned}
d \ln u_{t+1} &= -0.014 u_t - 0.94(r_t - d \ln p_{t+1}) - 0.54 \ln \omega_t + 0.12, \\
d \ln e_{t+1} &= 0.18 d \ln u_{t+1} \quad [\text{ or in integrated form: } e_t = u_t^{0.18}], \\
r_{t+1} &= 0.92 r_t + 0.06 d \ln p_{t+1} + 0.01 u_t - 0.01, \\
d \ln w_{t+1} &= 0.13 e_t - 0.07 \ln \omega_t + 0.49 d \ln p_{t+1} + 0.51 \pi_t^{12} - 0.12, \\
d \ln p_{t+1} &= 0.04 u_t + 0.05 \ln \omega_t + 0.18 d \ln w_{t+1} + 0.82 \pi_t^{12} - 0.04.
\end{aligned}$$

We have now one law of motion less than before, since the adaptive expectations mechanism (10) for the inflationary climate has been replaced here by a moving average expression, which—when translated back into such a mechanism—gives rise to an adjustment speed of approximately $\beta_{\pi^c} = 0.15$ in the law of motion for the inflation climate π^c. Taken together we therefore obtain the numerical specification (12)–(15) below for the reduced form 4D dynamics of Chen et al. (2006), where the measured form of Okun's law $e_t = u_t^{\beta_{e\hat{u}}} = u_t^{0.21}$ has been linearized around the steady state and used to eliminate e_t, and where the two linear structural equations for $d \ln w_{t+1} - \pi_t^{12}, d \ln p_{t+1} - \pi_t^{12}$ have been solved and substracted from each other in order to obtain the law of motion for $d \ln w_{t+1} - \pi_t^{12} - (d \ln p_{t+1} - \pi_t^{12}) = d \ln w_{t+1} - d \ln p_{t+1} = d \ln \omega_{t+1}$, solely as a function of the capacity utilization rates of firms and of workers and the current level of the real wage. The estimated system can thus be written:

$$\begin{aligned}
d \ln u_{t+1} &= -0.14 u_t - 0.94(r_t - d \ln p_{t+1}) - 0.54 \ln \omega_t + 0.12 & (12) \\
r_{t+1} &= 0.92 r_t + 0.06 d \ln p_{t+1} + 0.01 u_t - 0.01, & (13) \\
d \ln \omega_{t+1} &= -0.002 u_t - 0.089 \ln \omega_t + 0.061, & (14) \\
d \pi_t^c &= 0.15(d \ln p_{t+1} - \pi_t^c). & (15)
\end{aligned}$$

In order to get an autonomous system of difference equations in the state variables (capacity utilization u_t, the nominal rate of interest r_t, the real wage rate ω_t, and the

inflationary climate expression π_t^c) we have to insert into (12) and (13) the reduced form expression

$$d\ln p_{t+1} = \kappa[\beta_{pu}(u_t - 1) + \beta_{p\omega}\ln\omega_t + \kappa_p(\beta_{we}(e_t - 1) - \beta_{w\omega}\ln\omega_t)] + \pi_t^c,$$

for the PPC, which here simplifies to the expression

$$d\ln p_{t+1} = 0.08u_t + 0.03\ln\omega_t + \pi_t^c - 0.066, \tag{16}$$

when use is made again of Okun's law linearized around its steady state value 1. As noted, we have also made use of Okun's law in integrated form in the law of motion for real wages, see (9), which when inserted into it gives the parameter $\alpha = \kappa[(1 - \kappa_p)\beta_{we}\beta_{e\hat{u}} - (1 - \kappa_w)\beta_{pu}]$, in front of the rate of capacity utilization u_t (by which e_t has been replaced in the demand pressure term of the WPC). This parameter α is the critical condition for the working of the so-called Rose- or real-wage-effect, since when α is positive it states that real wage growth is positively correlated with economic activity and thus—due to the estimated form of the goods market dynamics, (in the process situation the law of motion for the rate of capacity utilization)—negatively responds to its level with a time delay, if this law of motion for u is also taken into account. Due to these two laws of motion and the size of their estimated coefficients, the role of income distribution in the fluctuations generated by the model does not appear to be an important one as far as the AD side of the model is concerned, since the aggregate influence of the rate of capacity utilization on real wages is a weak one (due to the estimated form of Okun's law). Note however that although it may be weak, this feedback chain from real wage changes via capacity utilization changes back to real wage changes is a destabilizing one.

The estimated system (12)–(16) however shows that from the partial perspective this destabilizing crossover feedback channel between real wage changes and rates of capacity utilization changes may be stabilized by the Blanchard and Katz error correction terms in the law of motion for real wages (though this latter effect concerns the trace of the Jacobian, while the former crossover effect concerns their minors of order two). Note however, that the dynamics of income distribution are here embedded in a framework with estimated parameter sizes that are held constant over the whole observation period, so that income distribution here works in a Keynesian environment with in particular rigid, and hence not systematically varying in time propensities to consume and invest, in contrast to what has been suggested by Keynes (1936) in his analysis of goods market dynamics in chapter 22. We therefore conclude that certain business cycle generators (accelerators) are still absent from the considered dynamics in their present form which implies that the implied phase length for the cycle will exceed considerably those actually observed for the U.S. economy, as is indeed seen in Figure 1.

We thus obtain from the reduced form representation (12)–(16) of our estimated DAS-DAD dynamics the result that the growth rates of u and ω both depend negatively in a cross-over fashion from each other, establishing a (weakly) destabilizing feedback chain between capacity utilization and real wage dynamics, or an adverse Rose-effect, if the parameters are taken just as estimated and confidence intervals still
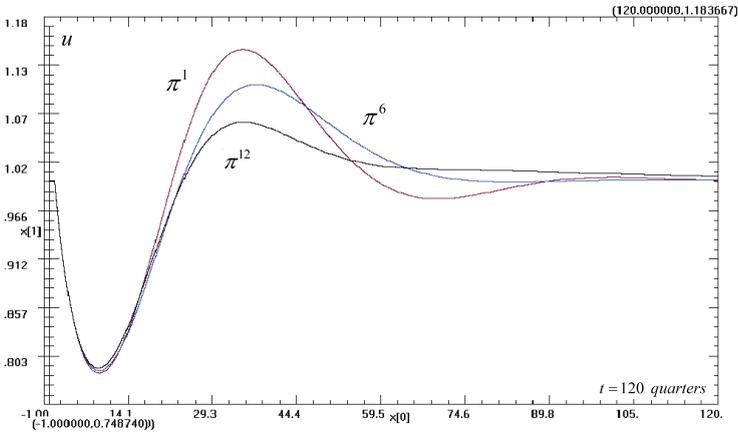
Note: Based on inflationary climate terms π^c with linearly declining weights with $m = 12, 6, 1$ quarter lengths.

Figure 1. Responses to positive real wage shocks for the three sets of estimated parameter values

neglected. This result is due to the fact that the apparently dominant effect of e on $\hat{\omega}$ is significantly reduced by the weak link between the rate of employment (a stock ratio) and the rate of capacity utilization (a flow ratio) established through our estimate of Okun's law.

Besides the Rose-effect we have the usual rate of interest channel in the law of motion for the rate of capacity utilization, whereby the central bank policy can influence the economy in a stabilizing fashion, which in fact is a substitute for the conventional Keynes-effect, but whereby also the destabilizing Mundell-effect comes into being. This latter effect establishes a positive link between the rate of capacity utilization and its rate of change, since the real rate of interest depends negatively on the inflation rate and thereby also negatively on the rate of capacity utilization. However, since we have estimated that the parameter β_{pu} is likely to be very small (implying that there is not a strong dependence of price inflation on demand pressure in the market for goods), the destabilizing Mundell-effect will be relatively weak, despite a significant negative dependence of the growth rate of economic activity on the real rate of interest.

We have furthermore from the partial perspective a stable dynamic multiplier and (as already noted) a stabilizing influence of real wages on their rate of change, established by the Blanchard and Katz (1999) error correction terms in the wage and price Phillips curves. There is finally a positive link between changes in the inflation climate and the rate of capacity utilization, since the rate of price inflation and thus the inflationary climate depend positively on the rate of capacity utilization and since (again via the real rate of interest channel) the rate of change of the capacity utilization of firms depends positively on the inflation climate surrounding the current evolution of the economy (see the reduced form (16) for the PPC as well as equations (15) and (12)). This supplements the finding that an increase in the reaction of wage inflation to demand pressure should contribute to stability, while the opposite is true for an increase in the

reaction of price inflation to their measures of demand pressure. Of course, all these statements are only partial in nature and may be falsified as intuitive guidelines for the systems' (in)stability if the eigenvalues of the Jacobian of the full dynamical system are calculated numerically, since all these feedback chains only appear in part of the minors that are to be considered in the Routh-Hurwitz conditions for local asymptotic stability.

It has been established in Chen et al. (2006), see also the next section, that the estimated sign structure always implies local asymptotic stability of the steady state if the reaction of price inflation to demand pressure is sufficiently small, if the inflationary climate is updated sufficiently slowly and if interest rate smoothing is sufficiently weak, but the reaction of the interest rate to inflationary gaps is sufficiently strong. Taking everything together, we therefore should expect convergence back to the steady state—in the case of a monetary policy that is sufficiently active when the considered dynamics are simulated numerically and shocked out of their steady state position.

This is indeed the case as is shown in Figure 1 (for a monetary policy that is sufficiently active) where we also show that this situation is not much changed if our twelve quarter moving average representation of the inflation climate is modified towards a six quarter moving average (again with linearly declining weights) and the model is reestimated. Similar observations also hold even in the case where only one quarter is considered, that is when the inflation climate is just represented by the inflation rate of the previous period as is usually the case in theoretical analyses of the New Keynesian approach augmented with the treatment of hybrid expectations (to which the concept of an inflation regime or climate is however then no longer associated).

We thus obtain as a first numerical result that the economy seems to be very robust in the absorption of supply-, demand-side and policy shocks (if indeed the reaction of the interest rate to the inflation gap is somewhat more active than estimated). In Figure 1 we exemplify this result through the application of a positive real-wage shock (caused by an increase in money wages or a decline in the general price level). The response is a significant decline in the rate of capacity utilization for approximately three years and then a slow and overshooting recovery over the next seven years until the economy starts converging back to its steady state position with more or less mild fluctuations. This result holds unambiguously for the inflation climate expressions $\pi_t^{12}, \pi_t^6, \pi_t^1$ discussed earlier. However the overshooting mechanism can be seen to become the stronger the faster the inflationary climate adjusts to the short-run fluctuations of the actual exchange rate.

It is somewhat perplexing to know from the theoretical analysis of the model that it loses its stability by way of a Hopf bifurcation[4] if the speed of adjustment of the inflationary climate becomes sufficiently fast, and to find empirically that convergence of the dynamics back to its steady state position is guaranteed even if only a one quarter lag applies for the representation of the inflation climate (which is then literally speaking no longer interpretable as a climate expression). We believe that this tension is basically due to the fact that in our estimation we did not allow for nonlinear behavioral relationships, a task that still remains to be undertaken. In view of the bounded

---

[4] In the case where $\gamma_{rr} < \gamma_{rp}$ holds; see our stability considerations below.
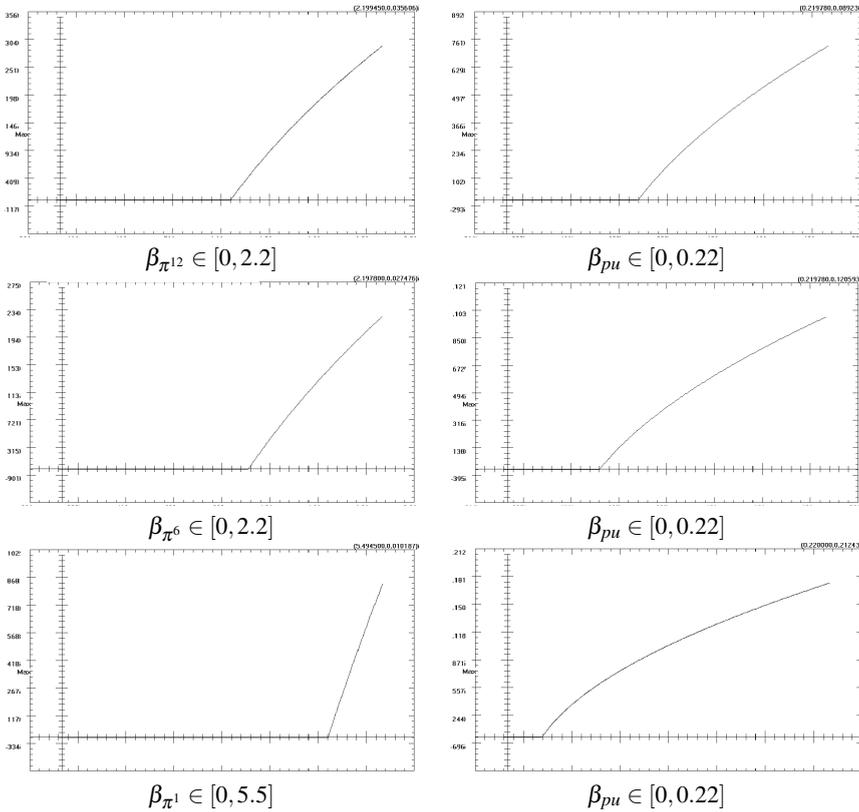
Figure 2. Eigenvalue diagrams for varying values of β_{pu} and $\beta_{\pi^{12}}$, β_{π^6}, β_{π^1}

fluctuations contained in the employed data set, it seems that the applied econometric methodology (see Chen et al. 2006 for details) and the by and large linear structure of the model basically prevent the establishment of dominant eigenvalues outside the unit circle (or a dominant positive real part in the continuous-time version of the model). The only weak evidence for an increased tendency towards instability is the increase in volatility that is shown in Figure 1 when the time horizon in the formation of the inflation climate expression becomes smaller and smaller. We note however again that the reaction of the Central Bank to the inflation gap must be increased to a certain degree as compared to the estimated value in order to remove a slightly positive real eigenvalue from the simulated continuous time dynamics.

In the eigenvalue diagrams shown in Figure 2, we see that for $\gamma_{rr} < \gamma_{rp}$, we in addition note the fact that the loss of stability by way of a Hopf-bifurcation (generally leading to the death of an unstable limit cycle around a stable corridor of the dynamics or the birth of a stable limit cycle after the loss of stability of the steady state) becomes more and more delayed if we use the estimated parameters for the cases π^{12}, π^6, π^1, in

this order, as shown in the left hand panels of Figure 2. Faster and faster adjustment of the inflationary climate expression in our continuous time version of the dynamics thus does not lead to a decrease in the Hopf bifurcation point, but to its increase instead. We note here that—due to the estimated form of Okun's law we have that one eigenvalue of the 5D dynamics must always be zero so that only the local instability range is clearly shown in the eigenvalue diagrams in Figure 2. The increased tendency towards instability, if we run through the sequence $\pi_t^{12}, \pi_t^6, \pi_t^1$, can be mirrored by our estimated models to a certain degree when we consider the same situation of a loss of stability by way of the parameter β_{pu} in the place of the parameter β_{π^c} as is shown in the right hand panels of Figure 2. Here it is shown that the interval for this parameter where stability prevails becomes smaller and smaller so that in the case of π^1 we even get loss of stability within the confidence interval for the estimated parameter. We therefore find, even in the case where behavioral nonlinearities are ignored in theory and in estimation, that the vulnerability of the dynamics towards the establishment of explosive adjustment processes becomes larger the faster the inflation climate is adjusting towards the actual course of price inflation.

In Chen et al. (2006) we found in addition that all partial feedback chains (including the working of the Blanchard and Katz error correction terms) translate themselves into corresponding 'normal' eigenvalue reaction patterns for the full 5D dynamics (with Okun's law added in its estimated derivative form), with the exception of the speed parameter β_{we} (the speed of adjustment of normal wages to the output gap in the labor market) where the eigenvalue analysis has shown that increasing wage flexibility may indeed become destabilizing if it is made very large. This provides one example of the situation in which partial economic insight can be misleading due to the fact that the corresponding feedback chain is only a small component of the many minors of the Jacobian of the dynamics at the steady state that have to be investigated in the application of the Routh-Hurwitz conditions to the full 4D dynamics (where Okun's law is applied in level form).

Increasing price flexibility has been found to be destabilizing, since the growth rate $\hat{e}$ of economic activity can thereby be made to depend positively on its level (via the real rate of interest channel, see (6)), leading to an unstable augmented dynamic multiplier process in the trace of the Jacobian J of the system under such circumstances. Furthermore, such increasing price flexibility will give rise to a negative dependence of the growth rate of the real wage on economic activity (whose rate of change in turn depends negatively on the real wage) and will thus lead to further sign changes in the Jacobian J. Increasing price flexibility is therefore bad for the stability of the considered dynamics from at least two perspectives. Nevertheless for the model as estimated there seems to be no problem for the working of the economy, since economic shocks may have long lasting consequences (due to our measurement of time-constant parameters) when sufficiently large, but are always absorbed by the economy through nearly monotonic adjustments, once the effect of the shock has become reversed.

The above impression may however be misleading if one further aspect of the functioning of actual market economies is taken into account (to which attention has not yet been paid in our estimation procedures). Hoogenveen and Kuipers (2000) have
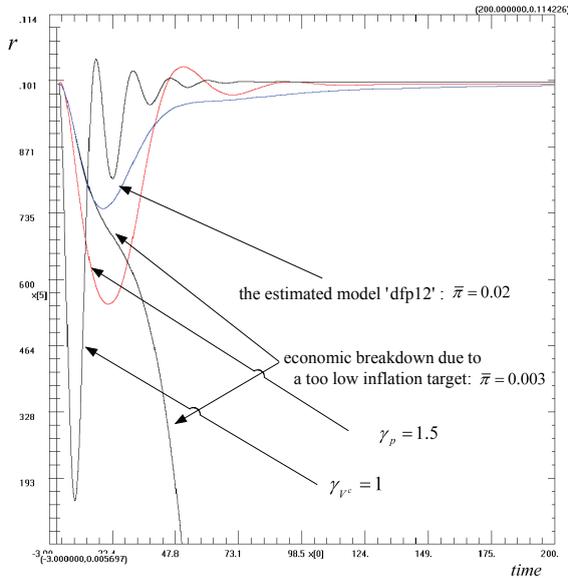
Figure 3. Downward money wage rigidity and too restrictive inflation targets

established for six European countries that money wages are not only completely rigid downwards, but in fact exhibit a floor for their rate of growth, which is bounded from below by a positive value. They basically therefore establish a kink in the WPC at positive rates of wage inflation. Chen and Flaschel (2005) do not find such a strong result in the case of the U.S. economy, but find also at least some evidence that money wages can be considered as being downwardly rigid. We thus now reconsider the above dynamical system for the new situation where wages can rise as specified, but cannot fall, that is, we exclude wage deflation from consideration. In such a case we can establish the results shown in Figure 3 where the estimated model and its shock absorber properties are repeated for the case π^{12} (with the inflation target of the central bank still set at two percent and again we observe convergence, with greater of less volatility depending on the values of γ_{rp} and γ_{ru}).

However convergence is lost if monetary policy is tighter in its inflation target, with the situation when $\bar{\pi} = 0.003$ also illustrated in Figure 3 and for which we see a radical change in the system's behavior. Since money wages cannot fall and since price can still fall in the depression generated by the assumed positive real wage shock, though price flexibility was very sluggish, we find in such a situation that the real wage must increase. This downward adjustment mechanism continues to work and it makes the depression deeper and deeper until the economy breaks down, exemplified in Figure 3 by means of the evolution of the nominal rate of interest. There exists therefore a great danger for systems with profit-led goods market dynamics, since downward price-flexibility coupled with downwardly rigid wages will then necessarily lead

the economy into a deflationary spiral when deflation comes about through shocks or other events. We also show in this figure how a more active interest rate policy with respect to the inflation as well as the output gap can avoid such a breakdown if it is set sufficiently strongly. However for our purposes here we only require a global floor to the evolution of the money wage (or its inflation rate) to find that a situation that initially appeared to be a very stable one without such a nonlinearity can turn out to be disastrous for the economy. The question therefore is by which mechanisms such a monotonic tendency towards more and more severe depressions can be stopped or even reversed. This question will be further pursued after the following section which is devoted to a stability analysis of the dynamics with the estimated parameter signs and the theoretical occurrence of instability if a floor to money wage inflation is added to the model.

4. Analyzing an estimated version of the model

Based on our estimates (12)–(15), the theoretical model (6)–(11) can now be simplified to the following qualitative format (with b set equal to $\beta_{e\hat{u}}$ for notational simplicity):

$$\hat{u} = -\alpha_{uu}(u-1) - \alpha_{ur}(r - \hat{p} - (r_o - \bar{\pi})) - \alpha_{u\omega} \ln \omega, \tag{17}$$

$$\dot{r} = -\gamma_{rr}(r - r_o) + \gamma_{rp}(\hat{p} - \bar{\pi}) + \gamma_{ru}(u-1), \tag{18}$$

$$\hat{\omega} = \kappa[(1 - \kappa_p)(\beta_{we}(u^b - 1) - \beta_{w\omega} \ln \omega) - (1 - \kappa_w)(\beta_{pu}(u-1) + \beta_{p\omega} \ln \omega)], \tag{19}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\hat{p} - \pi^c), \tag{20}$$

$$\hat{p} = \kappa[\beta_{pu}(u-1) + \beta_{p\omega} \ln \omega + \kappa_p(\beta_{we}(u^b - 1) - \beta_{w\omega} \ln \omega)] + \pi^c, \tag{21}$$

where the $\hat{p}$−equation has to be inserted again in some of the other equations in order to arrive at an autonomous system of differential equations. Doing this, rearranging items and linearizing around the steady state, then gives rise to:

$$
\begin{aligned}
\hat{u} &= -[\alpha_{uu} - \kappa(\beta_{pu} + \kappa_p\beta_{we}b)]u - \alpha_{ur}r - [\alpha_{u\omega} - \alpha_{ur}\kappa(\beta_{p\omega} - \kappa_p\beta_{w\omega})]\omega + \alpha_{ur}\pi^c + const \\
&= -a_1 u - a_2 r - a_3 \omega + a_4 \pi^c \pm a_o, \\
\dot{r} &= [\gamma_{rp}\kappa(\beta_{pu} + \kappa_p\beta_{we}b) + \gamma_{ru}]u - \gamma_{rr}r + \gamma_{rp}\kappa(\beta_{p\omega} - \kappa_p\beta_{w\omega})\omega + \gamma_{rp}\pi^c + const \\
&= +b_1 u - b_2 r \pm b_3 \omega + b_4 \pi^c \pm b_o, \\
\hat{\omega} &= \kappa([(1 - \kappa_p)\beta_{we}b - (1 - \kappa_w)\beta_{pu}]u - [(1 - \kappa_p)\beta_{w\omega} + (1 - \kappa_w)\beta_{p\omega}]\omega) + const \\
&= +c_1 u - c_3 \omega \pm c_o, \\
\dot{\pi}^c &= \beta_{\pi^c}\kappa[(\beta_{pu} + \kappa_p\beta_{we}b)u + \kappa(\beta_{p\omega} - \kappa_p\beta_{w\omega})\omega] + const \\
&= +d_1 u \pm d_3 \omega \pm d_o,
\end{aligned}
$$

where a_o, b_o, c_o and d_o subsume a number of constants.

Making use of our estimated parameter values we have assumed, on the one hand, in the law of motion for the rate of capacity utilization u that the $\hat{p}_u, \hat{p}_\omega$ components are dominated by the direct influences of u, ω on the growth rate of capacity utilization. In the law of motion for real wages on the other hand, we assume however, by way of

a slightly larger coefficient b, that the growth rate of real wages depends positively on the rate of capacity utilization (i.e., that β_{we} is the dominant term in this respect). This positive dependence may be a weak one, and also as before weakly negative, since our estimate of Okun's law implies in any case only a fairly weak impact effect of the capacity utilization rate on the rate of employment. We thereby get a sign structure for the partial derivatives of our dynamical system and thus its Jacobian J at the steady state, of the form

$$
J = \begin{pmatrix} - & - & - & + \\ + & - & \pm & + \\ + & 0 & - & 0 \\ + & 0 & \pm & 0 \end{pmatrix},
$$

where the $\pm$ items are solely due the two opposing real wage or Blanchard and Katz error correction terms in the reduced form PPC.

Proposition 1. *Assume that the sign structure of the matrix J applies and that its entry J_{23} is sufficiently small. Then the steady state of the dynamics (17)–(20), with (21) inserted into them, is locally asymptotically stable, if the inflationary climate expression π^c is updated in a sufficiently sluggish way, and if $\gamma_{rr} < \gamma_{rp}$ holds true.*

Proof. Let us first consider the case in which $\beta_{\pi^c} = 0$ holds true. We consider the submatrix J(3,3) for the remaining laws of motion, with J_{23} set equal to zero, which is then given by

$$
J(3,3) = \begin{pmatrix} - & - & - \\ + & - & 0 \\ + & 0 & - \end{pmatrix}.
$$

The characteristic polynomial of this matrix is

$$
p(\lambda) = \lambda^3 + k_1 \lambda^2 + k_2 \lambda + k_3, \text{ where } k_1 = -\text{trace } J(3,3), \ k_3 = -\det J(3,3),
$$

and it is easily shown to have only positive coefficients. Furthermore, the condition $k_1 k_2 - k_3 > 0$ is also fulfilled, since $\det J(3,3)$ is completely dominated by the expressions that make up the $k_1 k_2$ term. We thus have that the Routh-Hurwitz conditions for local asymptotic stability apply to the given situation, implying that the real parts of the eigenvalues of this polynomial must all be negative.

Consider now the case $\beta_{\pi^c} > 0$. We find for $\det J$ the sign structure

$$
\det J = \det \begin{pmatrix} - & - & - & + \\ - & - & 0 & + \\ + & 0 & - & 0 \\ + & 0 & 0 & 0 \end{pmatrix} = \det \begin{pmatrix} 0 & - & 0 & + \\ 0 & - & 0 & + \\ 0 & 0 & - & 0 \\ + & 0 & 0 & 0 \end{pmatrix},
$$

which implies a positive determinant given our assumption on the parameters γ_{rr} and γ_{rp}. Making the parameter β_{π^m} slightly positive moves the eigenvalues with three negative real parts and one zero eigenvalue slightly, such that they are now all in the negative half plane of the complex plane (since eigenvalues depend continuously on the parameters of the model and since the determinant is the product of all four eigenvalues). This implies the assertion of Proposition 1. □

Proposition 2. *Assume that the sign structure of the matrix J applies and that its entry J_{23} is sufficiently small.*

(i) *Increasing the adjustment speed β_{pu}, β_{wu} to a sufficient degree gives rise to a Hopf-bifurcation, via dynamic multiplier instability, where the system loses its asymptotic stability either accompanied by the death of an unstable limit cycle or the birth of a stable limit cycle.*

(ii) *Assume $\alpha_u r = 0$ in which case (i) does not apply. Then, increasing the adjustment speed β_{pu} to a sufficient degree again gives rise to a Hopf-bifurcation towards the above type of instability, now via an adverse real wage adjustment (an adverse Rose effect).*

(iii) *Increasing the adjustment speed β_{π^c} to a sufficient degree gives also rise to the above type of a Hopf-bifurcation, now via an adverse real interest rate adjustment (the so-called Mundell-effect).*

Proof.

(i) In this case the entry J_{11}, characterizing the overall effect of utilization changes on the growth rate of capacity utilization, becomes positive and can be made as large as needed in order to arrive at a positive trace of the matrix J. The Hopf-bifurcation then occurs when $k_1 k_2 - k_3$ becomes zero, which must be the case before trace $J = 0$ is established.

(ii) In this case J_{11} remains negative, while $J_{12}J_{21}$ is zero. The only entry in the coefficient k_2 that then depends on the parameter β_{pu} is then given by $J_{13}J_{31}$, where J_{31} must become positive for increasing β_{pu}, while J_{13} remains negative, thereby establishing a positive feedback channel between capacity utilization and real wages that makes k_2 negative if β_{pu} becomes sufficiently large. The Hopf-bifurcation then occurs when $k_1 k_2 - k_3$ becomes zero, which must be the case before $k_2 = 0$ is established.

(iii) Obviously, since the parameter β_{π^c} only appears in the product $J_{14}J_{41}$ which establishes a positive link between the evolution of the inflation climate π^c and capacity utilization u, the destabilizing Mundell-effect of conventional macrodynamic model building. $\qquad \square$

Proposition 3. *Assume that the sign structure of the matrix J applies. Assume furthermore that monetary policy is impotent by setting $\alpha_{ur} = 0$. Assume finally that the negative Blanchard and Katz error correction mechanism in the real wage dynamics is sufficiently weak (i.e., the term $(1 - \kappa_w)\beta_{p\omega}$ is chosen sufficiently small). Then, the steady state of the considered dynamical system is unstable in the downward direction, if there is a global floor to money wage inflation at its steady state value, i.e., any initial and contractive u or ω shock will then lead to an ever accelerating contraction of the economy.*

Proof. In the considered situation, the interacting dynamics are reduced to a 2D dynamical system in the rates u and ω. This system exhibits a negative determinant of its Jacobian at the steady state $u_o = 1$, $\omega_o = 1$ and two 1D stable manifolds that cannot

be reached by contractions in u or expansions in ω. This implies that such shocks always lead to trajectories with a declining rate of capacity utilization along them. □

If monetary policy is only weakly influencing the private sector and if the Blanchard and Katz real wage error correction mechanism in the PPC is weak, then we have that the reaction of price levels to their corresponding demand pressure item and the missing reaction of wages in this regard will allow for an adverse adjustment, i.e., an increase in real wages that will lead the economy into deeper and deeper depressions. The question then is whether interest rate effects in goods demand and interest rate steering by the Central Bank can help to avoid such an outcome, since of course the destabilizing Mundell effect will then also be back in the feedback interactions of our economy, or whether monetary policy needs a systematic overhaul in a situation where there is an adverse real wage effect at work. Our analytical findings in the estimated situation thus are that the economy may work like a shock absorber for certain ranges of its parameter values, but that this property may get lost in a variety of ways, which then demand the introduction of further behavioral nonlinearities that can keep the resulting dynamics bounded despite the existence of centrifugal forces around its steady state position.

Our approach thus demands further behavioral nonlinearities in the case of explosive downward (or upward) business fluctuations, but not for the New Keynesian imposition of a mathematical boundedness condition that—if determinate—keeps the dynamics always on its stable arm, a result that is not at all in line with Keynes' (1936) own analysis of the trade cycle mechanism; see his Chapter 22 and the uses he makes of his three central parameters, the marginal propensity to consume, the marginal efficiency of investment and the state of liquidity preference. However, concerning our own approach, we find that the results on various degrees of downward money wage rigidities do not unambiguously support Keynes' view that workers' resistance against money wage reductions is always good for economic stability. Concentrating on monetary policy solely, the question in our case is whether a policy rule can be found that can tame the outbreak of accelerating downward or upward wage-price spirals and other explosive feedback mechanisms, while in the New Keynesian case one has to modify the dynamics such that there is a single bounded response in the considered 4D case or in higher dimensions if interest rate policy gives rise to such an increase in dimension.

5. Midrange downward money wage rigidity

In this section we show by means of a numerical example that the problematic downward rigidity of money wages may to some extent be needed in order to stabilize the economy when it is subject to explosive fluctuations caused by an increase in the speed with which the inflationary climate expression is adjusted. This rigidity should not however be global in nature, but give way again to downward wage flexibility if the rate of employment becomes sufficiently low. This particular, not implausible mix of different wage inflation regimes may be surprising with respect to its stability implications, but comes at least not completely unmotivated, due to the fact that wage flexibility tends to be stabilizing and price flexibility destabilizing from the partial perspective

of the real wage channel discussed above.

In order to formalize the envisaged three regimes of wage inflation needed for our subsequent simulations we make use of the three alternatives shown in Figure 4 for our reformulation of the WPC, from which the dynamics of real wages and price inflation in their reduced form presentation must now be derived. Underlying these three situations is the assumption that wages behave as in the original model considered in Sections 2 and 3 when their rate of change is above a certain floor f and when the rate of employment is above a certain critical level $\underline{e}$ below which workers will again accept faster decreases in their money wages than just the level f. Wage inflation is in the latter case assumed to be driven by $\hat{w} = \beta_{we}(e - 1) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c$, in the place of the working of the Phillips curve in normal or overheated situations, $\hat{w} = \beta_{we}(e - 1) - \beta_{w\omega} \ln \omega + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c$. In between we will find—as stated—a regime where wage inflation (or deflation) is just given by a rate f.

These three scenarios translate themselves into reduced-form real wage and price level dynamics as follows, where we now denote by $\hat{w}_{red}$ the reduced form money
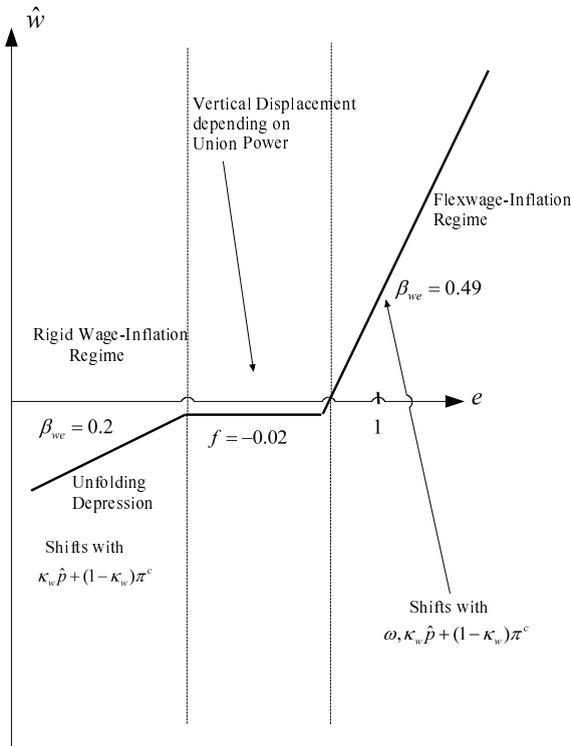
Figure 4. Three possible regimes for wage inflation

wage Phillips curve of the original model:

$$\hat{w}_{red} = \kappa[\beta_{we}e - \beta_{w\omega}\ln\omega + \kappa_w(\beta_{pu}u + \beta_{p\omega}\ln\omega)] + \pi^c.$$

If $\hat{w}_{red} < f$, $e \geq \underline{e}$, then

$$\hat{\omega} = (1 - \kappa_p)(f - \pi^c) - \beta_{pu}u + \beta_{p\omega}\ln\omega, \tag{22}$$
$$\hat{p} = \pi^c + \kappa_p(f - \pi^c) + \beta_{pu}u + \beta_{p\omega}\ln\omega. \tag{23}$$

If $\hat{w}_{red} \geq f$, $e \geq \underline{e}$, then the original dynamics (9), (11), applies while in all other situations we have to apply the equations

$$\hat{\omega} = \kappa[(1 - \kappa_p)\beta_{we}e - (1 - \kappa_w)(\beta_{pu}u + \beta_{p\omega}\ln\omega)], \tag{24}$$
$$\hat{p} = \kappa[\beta_{pu}u + \beta_{p\omega}\ln\omega + \kappa_p\beta_{we}(e) + \pi^c. \tag{25}$$

The money wage behavior underlying these modified dynamics is summarized in Figure 4. This situation represents an appropriate modification of Filardo's (1998) empirical analysis of such a type of Phillips curve, where however the price inflation gap (with respect to expected inflation) is used on the vertical axis. There is thus some evidence for such a three regime PC, though in a somewhat different context.

When one applies this type of nonlinear WPC to our semi-structural model in annualized form, here from this perspective with a tight inflation target of the central bank of $\bar{\pi} = 0.0077$, one gets the hierarchy of events shown in Figure 5. In situation 1, where there is no nonlinearity in the WPC, we still have the convergent result of Figure 2 (a positive real wage shock of 4 percent is here applied), though indeed the inflation target is now somewhat below the floor of wage inflation. In situation 2, we have that the floor $f = 0.01$ applies globally and get again economic breakdown
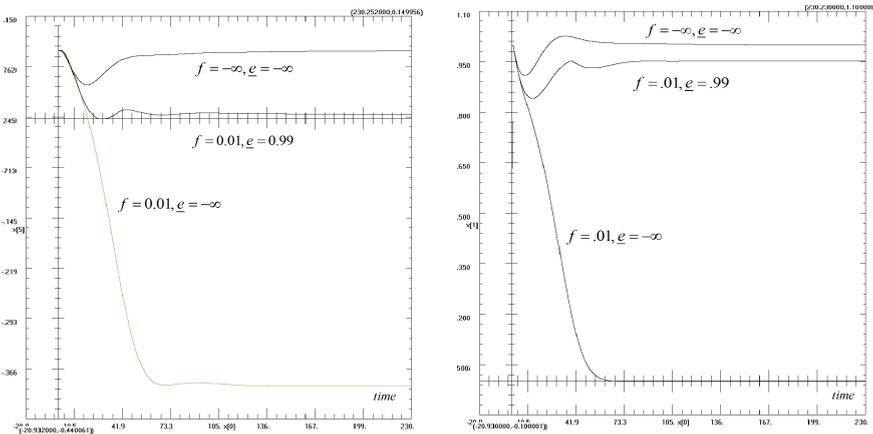
Figure 5. Convergent unrestricted dynamics and the role of piecewise nonlinear WPC's ($\bar{\pi} = 0.0077$)

due to the adverse working of the Rose real wage effect. In situation 3 where wage flexibility becomes established again with the same parameter value β_{we} (already below employment rates $e = 0.99$) we get an intermediate situation in which the rate of employment stays below 0.99, the rate of capacity utilization converges approximately to the value 0.95, and where there is ongoing deflation at the rate of -0.01. The real wage however stays 2 percent above its original steady state value and the nominal rate of interest remains positive, but is close to zero. The long-run of the model therefore departs significantly from the steady state values of the unrestricted dynamics with its linear WPC. From this example we therefore obtain the result that the three-regime WPC is better than the one with a global floor to money wage inflation, but the completely unrestricted model with its linear WPC still provides the best outcome after a contractionary real wage shock has hit the economy in these three scenarios.

The example just discussed applies to the situation where the adjustment of the inflationary climate is still sufficiently sluggish to guarantee the stability properties shown in Figure 1. The results obtained thus characterize an economy that exhibits strong convergence back to the originally given steady state if not restricted by behavioral nonlinearities of the discussed type. Let us next investigate a situation where the economy is destabilized by a change in the speed of adjustment of the inflationary climate that is surrounding it. We now assume in the place of the value, 0.15, the value, 1.52, for the parameter β_{π^c} in the considered situation. The result, in terms of capacity utilization, is shown in Figure 6 by the cyclical time series with symmetrically and
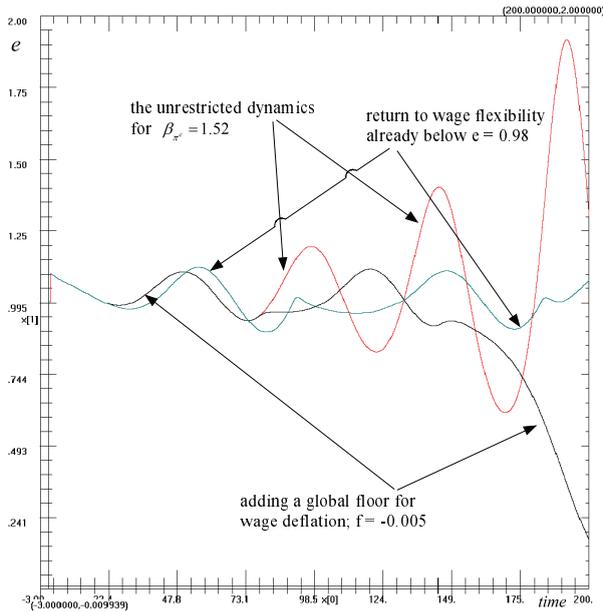
Figure 6. The role of regime changes in wage inflation dynamics

rapidly increasing amplitudes of the cycle.

Figure 6 therefore shows in the completely unrestricted case a time series for the rate of capacity utilization that will sooner or later lead to economic collapse if there is no change in the behavior of the economy. Adding now a global floor of $f = -0.005$ to the dynamics in the WPC component does not improve this situation, but leads again to monotonic economic breakdown instead. However if we allow for a third regime as described in Figure 4 on its left hand side, we then get evolutions of the state variables of the model that remain bounded to an economically meaningful domain. In addition, these dynamics are now of a mathematically complex type (but only somewhat irregular from the economic point of view) as is shown in Chen et al. (2006) who investigate the stable trajectory shown in Figure 6 in more detail.

With the assumed change in the adjustment speed of the inflationary climate expression the economy is therefore no longer viable in the long run (but cyclically explosive) and it becomes even less viable if a global floor $f = -0.005$ is introduced into the estimated WPC as shown in Figure 5. Yet assuming a three-regime WPC as discussed in connection with Figure 4 overcomes not only this latter monotonic downturn, but also the explosive fluctuations of the unrestricted case. Some downward flexibility of money wages in a certain midrange interval, giving way again to downward flexibility of money wages (at two percent rate of unemployment) provides viability to the evolution of the trajectories of the dynamics as indicated in Figure 6, here over a 50 year horizon.

6. Wage-led economies, increasing adjustment speeds and the emergence of complex dynamics

In this section we provide some further simulations of the general model with admissible, but no longer estimated parameter values in order to also consider in particular a situation where the Rose effect works in the opposite way, i.e., where aggregate demand and the adjustment of the rate of capacity utilization depend positively on the real wage and where therefore wage flexibility with respect to demand pressure on the labor market should be destabilizing. In such a situation a global floor to wage inflation or deflation should therefore save the economy from economic breakdown, since downwardly rigid money wages combined with downwardly (somewhat) flexible prices leads to an increase in real wages which in the present situation stimulates economic activity and thus should lead the economy out of the depression. Yet, due to the fact that the unrestricted economy is here strongly explosive, we find after each recovery that explosive forces come into being each time in a somewhat modified manner, until the economy falls back again into a depression with downward money wage rigidity avoiding its further destabilization until a new recovery sets in.

The base parameter set underlying the simulations shown below is shown in Table 1. Speeds of price and wage adjustment are somewhat higher, while the adjustment speed of the inflationary climate is only 1/3 of the value used in the preceding section. We have a positive real wage effect in the goods market dynamics (a wage-led regime now) and have now also included a negative real wage effect in the law of motion of

the rate of employment, which furthermore now depends on the level of the rate of capacity utilization in addition to the growth rate of capacity utilization we have used so far as the sole determinant of rate of employment changes. The rate of employment is thus no longer strictly positively correlated with the rate of capacity utilization as was the case in our estimate of the model, which makes the critical α condition considered in Section 3 more difficult to obtain though of course wage flexibility must now be destabilizing). Finally, monetary policy is now more active with respect to the state of the business cycle and we have a floor to wage deflation that is practically zero. The result of this combination of parameter values will be, as is shown below, that the nonlinear WPC of Figure 2, now with a global floor ($\underline{e} = 0$)—-in fact the only important nonlinearity in our 5D dynamical system—is capable of keeping the highly explosive unrestricted dynamics within economically meaningful bounds. This result is achieved in a way that makes the resulting attractors complex from the mathematical perspective, though not too irregular from the economic perspective.

Table 1. Baseline parameters set for the wage-led economy simulations

$\beta_{pu} = 1$	$\beta_{p\omega} = 0.4$	$\kappa_p = 0.3$	$\beta_{we} = 0.8$	$\beta_{w\omega} = 0.4$	$\kappa_w = 0.7$	$\beta_{\pi^c} = 0.5$
$\alpha_{uu} = 0.22$	$\alpha_{u\omega} = -0.1$	$\alpha_{ur} = 0.25$	$\beta_{eu} = 0.15$	$\beta_{e\hat{u}} = 0.5$	$\beta_{e\omega} = 0.5$	
$\gamma_{rr} = 0.1$	$\gamma_{rp} = 0.5$	$\gamma_{ru} = 1$	$f = -0.0001$	$\underline{e} = 0$	$\omega_{shock} = 1.01$	

We illustrate the complex dynamics that are generated by this specific parameter set at first by showing in Figure 7 projections of its now 5D format (since the rate of employment is now moving independently from the rate of capacity utilization to some extent) into 2D subplanes of the full 5D phase space. We see top left in Figure 7 the partial phase plot of the real wage against the rate of employment with a by and large clockwise orientation and phase length corresponding to what is known on the Goodwin (1967) 2D growth cycle model and its empirical analogue. In periods of high employment however this clockwise movement of these two state variables gives way to some local and fast fluctuations which represent the explosive part of the dynamics. Below this figure we see the projection of the attractor into the rate of employment/inflation climate subspace where we would expect—due to what is known from unemployment—inflation phase plots (and their clockwise orientation)—a counterclockwise orientation which is not clearly visible there. This orientation is however typical for the projection of the attractor top right, there for a capacity utilization rate and inflation climate subspace projection. We see there too, that when recovery sets in, the dynamics are squeezed through a small corridor or eye of a needle followed by a small stagflation cycle, which is then followed by a large cycle until the economy is squeezed back into the small corridor for its next upswing phase. The time series bottom-right adds to this the information that the sequence of business fluctuations generated by the present parameter set is irregular and regular at one and the same time, irregular from the mathematical point of view in its amplitudes and regular from the economic point of view due to the repetitive behavior in the succession of small and large cycle patterns.
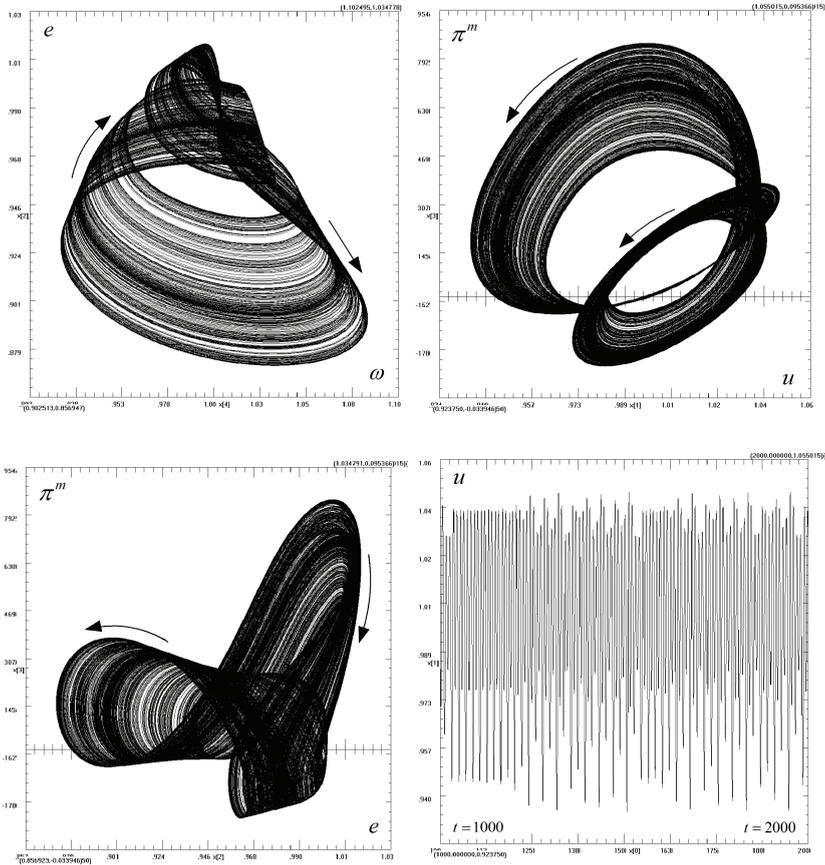
Figure 7. Projections and a time series representation of the attractor of the dynamics of the wage-led economy

In Figure 8 we present some bifurcation diagrams around the set of parameter values of this section showing the plots of local maxima and minima (in the vertical direction) plotted against one typical parameter on the horizontal axis. The figures show broad bands where these minima and maxima are (fairly) dense and at the end of the shown parameter ranges, or in between, limit cycle behavior which to some extent exhibits situations of period doubling routes to complex dynamics. Top left we have plotted the rate of capacity utilization against the parameter $\alpha_{u\omega}$ which determines the strength of the impact of real wages on goods market evolution. Top right the capacity utilization rate is plotted against the policy parameter γ_{ru} which determines the strength of the reaction of the Central Bank to the activity level of the economy (a parameter that is normally not so much at the centre of interest, as is the one in front of the inflation gap). We can see there that positive values of $-\alpha_{u\omega}$ (profit-led regimes) and
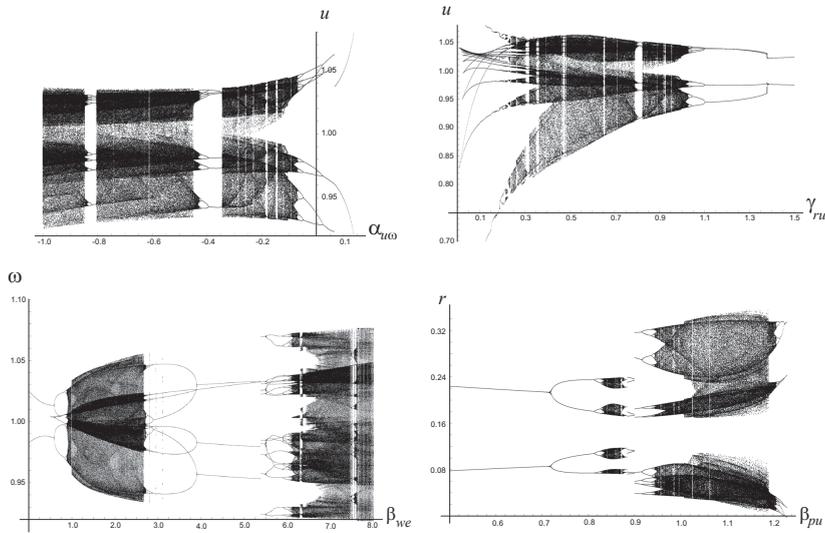
Figure 8. Bifurcation diagrams for selected adjustment speeds and behavioral coefficients of the dynamics of the wage-led economy

low γ_{ru} create viability problems for the considered economy. Note that we measure the positive effect of real wages on the growth rate of capacity utilization by means of negative numbers, since the negative effect was measured by a positive number in our estimates. Due to the working of the global floor on money wage deflation we have a fairly stable corridor within which the rate of capacity utilization is fluctuating for a large domain of $\alpha_{u\omega}$ values, which is not true in a similar way for variations in the policy parameter γ_{ru}. In the latter case the complex dynamics can even be made to disappear as outcome if the parameter γ_{ru} is increased a little bit beyond 1, the parameter value we used to generate Figure 7 and its complex dynamics. There are also windows in the case of the parameter $\alpha_{u\omega}$ which however disappear again if the parameter is set to even larger values.

Bottom left in Figure 8 we see (for $\beta_{pu} = 0.8$ in the place of $\beta_{pu} = 1$) the local maxima and minima of real wages plotted against the speed of adjustment of money wages with respect to demand pressure in the market for labor. We have again a fairly stable corridor (enclosed by the interval $(0.9, 1.1)$ within which the real wage is moving when the parameter is increased from close to zero up to eight. We have complex dynamics at the point which corresponds to the above parameter set with increases in maximum amplitudes thereafter, but with a sudden return to limit cycle behavior at approximately $\beta_{we} = 2.6$. Fluctuations thereafter even become less pronounced until there is again a period doubling sequence back to complex dynamics. There is therefore no global property that the considered wage adjustment speed is stabilizing in the sense of eigenvalue analysis as one would expect it from the partial reasoning concerning the

real-wage channel in a wage-led regime. Bottom-right finally we show the fluctuations in the nominal rate of interest (their local maxima and minima) plotted against the speed of adjustment of price inflation with respect to demand pressure in the market for goods. The system bifurcates systematically into complex dynamics as this speed of adjustment is increased, yet it is still kept viable over a range 0.95–1.2 approximately.

To summarize we therefore have found that increasing wage and price flexibility does not appear to be good for economic stability, even if tamed by a rigid floor to money wage declines, while the role of income distribution in the determination of goods market dynamics appears to be similar over wide ranges of the parameter $\alpha_{u\omega}$, though interrupted by phases of less complex dynamics (limit cycle behavior) and with tendencies towards instability if the economy switches from a wage-led regime to a profit-led one (where the impact of real wages on the growth rate of capacity utilization becomes a negative one). Finally, monetary policy that gives more and more weight to the state of the business cycle (as measured by u) becomes more and more stabilizing in the considered situation. It is obvious from these numerical simulations that the assumed kink in the WPC is of great importance for the behavior of the considered economy, since it makes an unstable wage-led regime a viable one and shapes the overall evolution of the economy in a way that cannot be discussed in terms of local stability analysis as in the New Keynesian model with which we have contrasted our DAS-DAD model in various sections of the paper.

7. Conclusions

We conclude that higher dimensional models can generate interesting patterns of business fluctuations (even for parameter ranges that correspond to empirically observed parameter sizes), and this in continuous time, where overly strong convergence properties do not give rise to overadjustment and instability as in discrete time systems. The complex dynamics in the model of this paper were not the outcome of such destabilizing overshooting and/or implausible and artificial nonlinearities as is often the case in much literature on chaotic dynamics. It was instead the plausible by-product of the combination of, by and large conventional (though not very often considered), higher dimensional Keynesian D(isequilibrium)AS-D(is-equilibrium)AD analysis with an important factual institutional nonlinearity, here stylized in the form of a kinked money wage Phillips curve (with one or even two kinks). In the present paper, this latter outcome was primarily shown for a situation where the goods market was wage-led, whereas Chen et al. (2006) investigated the same issue for the profit-led case, again in the case of a double-kinked WPC as estimated in Filardo (1998).

Compared to the outcomes found from the analysis of our model, the New Keynesian approach in its pure, microfounded form faces some significant difficulties. On the one hand, it is at odds with the facts even in its baseline form, see Mankiw (2001), since its type of forward-looking behavior leads to decelerationist PC's in the place of accelerationist ones. On the other hand, it must provide in the case of staggered wage and price setting complete instability of steady state solutions in order to allow for an appropriate application of the rational expectations (jump variable) methodology upon

which it is heavily reliant. This type of instability is however not easily established, since a conventional type of Taylor rule is no longer sufficient to guarantee such an outcome, even when made as active as is sensibly possible.

References

Asada, T., Chen, P., Chiarella, C. and Flaschel, P. (2006). AD-AS and the Phillips Curve. A Baseline Disequilibrium Model. In Chiarella, C., Flaschel, P., Franke, R. and Semmler, W. (eds.), *Quantitative and Empirical Analysis of Nonlinear Dynamic Macromodels. Contributions to Economic Analysis, vol. 277*. Amsterdam, Elsevier, 173–227.

Blanchard, O. J. and Katz, L. (1999). Wage dynamics: Reconciling Theory and Evidence. *American Economic Review*, 89(2), 69–74.

Chen, P., Chiarella, C., Flaschel, P. and Semmler, W. (2006). Keynesian Macrodynamics and the Phillips Curve. An Estimated Baseline Macromodel for the U.S. Economy. In Chiarella, C., Flaschel, P., Franke, R. and Semmler, W. (eds.), *Quantitative and Empirical Analysis of Nonlinear Dynamic Macromodels. Contributions to Economic Analysis, vol. 277*. Amsterdam, Elsevier, 229–284.

Chen, P. and Flaschel, P. (2005). Keynesian Dynamics and the Wage-Price Spiral. Identifying Downward Rigidities. *Computational Economics*, 25, 115–142.

Chiarella, C. and Flaschel, P. (1996). Real and Monetary Cycles in Models of Keynes-Wicksell Type. *Journal of Economic Behaviour and Organization*, 30, 327–351.

Chiarella, C., Flaschel, P. and Franke, R. (2005). *Foundations for a Disequilibrium Theory of the Business Cycle. Quantitative Analysis and Qualitative Assessment*. Cambridge, Cambridge University Press.

Filardo, A. (1998). New Evidence on the Output Cost of Fighting Inflation. *Economic Review*, QIII, 33–61.

Goodwin, R. M. (1967). A growth cycle. In Feinstein, C. H. (ed.), *Socialism, Capitalism and Economic Growth*. Cambridge, Cambridge University Press, 54–58.

Hoogenveen, V. C. and Kuipers, S. K. (2000). The Long-Run Effects of Low Inflation Rates. *Banca Nazionale del Lavoro Quarterly Review*, 53, 267–286.

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. New York, Macmillan.

Mankiw, G. (2001). The Inexorable and Mysterious Tradeoff between Inflation and Unemployment. *Economic Journal*, 111, 45–61.

Rose, H. (1967). On the Non-Linear Theory of the Employment Cycle. *Review of Economic Studies*, 34, 153–173.

Rosser Jr., J. B. (1999). On the Complexities of Complex Economic Dynamics. *Journal of Economic Perspectives*, 13, 169–192.

Rosser Jr., J. B. (2000). *From Catastrophe to Chaos: A General Theory of Economic Discontinuities, Vol. I. Mathematics, Microeconomics, Macroeconomics, and Finance*. Boston, Kluwer.

Woodford, M. (2003). *Interest and Prices*. Princeton, Princeton University Press.