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On the Essential Multidimensionality of an Economic Problem: Towards Tradeoffs-Free Economics

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Abstract The foundation of welfare economics is the assumption of Pareto-efficiency and its concept of tradeoffs. Also the production possibility frontier, efficiency frontier, nondominated set, etc., belong to the plethora of tools derived from the Pareto principle. The assumption of tradeoffs does not address the issue of system design or redesign in order to reduce or eliminate tradeoffs as a sure characteristic of suboptimal, inefficient system configuration. In this paper we establish that tradeoffs are not attributes of objectives, criteria or dimensions, as it is habitually assumed, but are the properties of the very sets of possibilities, alternatives or options they purport to value and measure. We use De novo programming, through which the so called feasible set of opportunities can be redefined towards optimal, tradeoffs-free configuration. The implications of tradeoff-free economics are too vast to foresee and elaborate in a single paper; they do touch the very foundations of economic thought. Some numerical examples are given in order to illustrate system-design calculations in linear systems.

Keywords Tradeoffs, multiple criteria, decision making, tradeoffs-free, optimization, De novo programming, Pareto-efficiency, added value
JEL classification C02, C61, D21, D61, D83, L21

1. Introduction

"An economic problem exists whenever scarce means are used to satisfy alternative ends. If the means are not scarce, there is no problem at all; there is Nirvana. If the means are scarce and there is only a single end, the problem of how to use the means is a technological problem. No value judgments enter into its solution; only knowledge of physical and technical relationships."

M. Friedman (1962: 6)

The 20th century was an era of tradeoffs: always giving up something in order to get something else, rarely attempting to aspire for multidimensional improvement. After the crisis of 2008, we are posed to move beyond the zero-sum, tradeoffs-based economy, i.e., from transferring wealth "from one pocket to another" (in a win-lose fashion), not creating value for both sides of a *transaction*. We expect to move towards a nonzero-sum economy where both sides benefit simultaneously (in a win-win

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fashion). Although general growth and progress "lifts all boats", the transactional economy rests on the tenuous assumption of economic tradeoffs between agents, products or dimensions, re-creating recursively zero-sum macro- and micro-conditions at different levels of "progress". Consumers (contrary to producers) *never* prefer tradeoffs but seek tradeoffs-free (or close to tradeoffs-free) alternatives, products or services, in order to realize added value through free-market transactions. It is this last problem — the shift from the tradeoffs-based to tradeoffs-free choices, decisions, strategies, and economics — that is the subject of the current paper.

We start with the quote from Milton Friedman: it was his transforming thought that stimulated this author to pursue problems of multiple criteria decision making (MCDM), Tradeoffs-free economics (TFE), and De novo programming (DNP) and Knowledge management (KM). In this paper we focus on developing the argument on the nature of tradeoffs and outline the foundations of tradeoffs-free economics.[1]

An *economic problem* is characterized by pursuing *multiple criteria* over a constrained set of decision alternatives. With the pursuit of multiplicity, *tradeoffs* emerge and *value judgments* must be exercised.

A *technological problem* is characterized by pursuing a *single criterion* over a constrained set of decision alternatives: *no tradeoffs* are possible (along any single dimension),[2] *no value judgments* enter, technical *computation* is sufficient.

Friedman was quite clear about what is the subject of economics: an economic problem — and that of engineering: a technological problem. It is social, economic and financial "engineering" that is at the core of the worldwide economic crisis.

It is important to note that Friedman's "single end" refers to *any* single end: be it an aggregate function, production function, utility function or any other macro-formula or composite of initially separate components. A single criterion or objective does not admit tradeoffs by definition. A necessary condition for tradeoffs (and value judgments) is the presence of multiple criteria (or multiple agents) in a *vector or portfolio sense*, not in a single-aggregate sense.

In this paper we argue against and beyond Friedman in a number of ways:

(i) The "technological problem" is still the domain of aggregation and utility-based macroeconomics.

(ii) Engineering itself has moved from the "technological problem" towards the "economic problem" via multiple criteria based design.

(iii) The "economic problem" has to be extended *beyond* optimization of the given, towards designing the optimal.

(iv) There are two distinct concepts of "profit maximization" in economics: maximizing profit with respect to a given system of resources and — designing the profit-maximizing system of resources.

[1] Friedman was primarily a micro-economist: his formative *Price Theory: A Provisional Text*, Chicago, IL: Aldine, 1962, was a staple of our PhD pursuits at the University of Rochester. The messianic and failed monetary theory and unregulated-market crusades were the products of his later years.

[2] Except for now fashionable bankruptcies, where one can be enticed to "trade off" a dollar (invested) for, say, fifteen cents (returned). Such "swaps" of accepting less for more are gambling, *not* economic decisions.

Understanding the role of tradeoffs in economics is not limited to their *a priori* acceptance (description-computation-analysis) but should include *designing* tradeoffs configuration and their ultimate elimination (prescription-design-synthesis). The role of economics is not to assume and describe tradeoffs, but to explore and design their optimal role in decision making and profit maximization of individuals and firms.

It is quite clear that customers and consumers do not like, do not want and do not need tradeoffs. There is no economic theory which would postulate human preferences for tradeoffs. Tradeoffs are forced on consumers as a fact or "necessity of life" and they are accepted only as such. Tradeoffs are the producer's concept (so called ***Or*** strategy), brought forth through concentrating on the "technological problem", i.e. one "key" dimension of production (cost *or* quality *or* speed *or*...) at a time, forcing consumers into tradeoffs when choosing a product. Tradeoffs are not the customer's concept (so called ***And*** strategy) where the focus is clearly on the "economic problem", i.e. all the "relevant" dimensions of a product (cost *and* quality *and* speed *and*...).

The tradeoffs-free option is always *demanded* while the tradeoffs-based option has always been *supplied*. The producer's strategy is traditionally ***Or***, while the consumer's strategy is always ***And***. This *preferential dichotomy* has already been changing in practice; it is now going to change in lagging theory as well.

2. Conventional allocation of resources

The main economic problem is clearly the use of limited resources to obtain certain economic *objectives*. Linear systems (i.e. fixed production coefficients), especially *linear programming* (LP), represent a good way of demonstrating the optimal allocation of resources.

The traditional form of LP can be summarized as follows: (i) Fixed technological coefficients, making up a matrix; (ii) Resources available specified, as a vector of amounts of primary factors which cannot be exceeded; (iii) The objective specified in terms of final products, together with the optimal algorithm for selecting the most efficient allocation of resources under the given technological conditions.

One can see that economic considerations were significantly compromised to fit mathematical limits of computing practices of central planning, with no room for markets, market prices and profits. Linear programming then has an obvious and direct bearing on the economics of socialism and collectivist planning (von Mises 1935).

Observe that most decisions in LP cannot be computed, but must be taken by someone, most likely by the state in the macro or by the planning body in the micro. Available resources must be specified, market prices of resources are not considered, and multiple criteria have been conveniently replaced by *a single objective superfunction* (to be also specified).

Koopmans (1951) introduced the so called *activity analysis* which reinforces the shortcomings of the LP model by stipulating that primary factors are available as resources *from outside the system considered*, designated as *given resources*.

Within given resources, a feasible allocation of resources is deemed *efficient* if an increase in one output can be achieved only at the cost of a decrease in another output. At an efficient point, one final product can be increased only at the expense of the reduced output of another final product or at the expense of the increased use of some input. In this very definition of the economic efficiency we find a built-in notion of the tradeoff. If both products were increased at the same time, the same solution would be deemed *inefficient*. The external fixation of resources is implicit and axiomatic. Feasible points lie in or on a *fixed* and invariant convex cone in m dimensions (m products). *Who* gives this "given" cone? And *how*?

It is logical that an efficient point (solution) must lie on the boundary of a feasible set. Therefore, the boundary itself must be fixed together with its tradeoffs. In the solution of the technological problem of resource allocation, given the technology and the criterion of efficient allocation, we search to distinguish between efficient and non-efficient allocations.

Also Samuelson (1965) advances the same simplification in a similar vein. He studies the maximum amount of output which can be produced from any *given* set of inputs, in order to establish the so called *production function*. Again, who gives or how these inputs are given is not addressed. He even assumes that such function is single-valued, just because it can have a desired continuous partial derivative of needed order. Samuelson still deals with the technological — not economic — problem in the sense of Friedman. Even profit is defined as the difference between gross revenue and total expenditure, where total expenditure is based on a *given* production function and *given* inputs. If so many things are *given* in economics (even though not in economy), then asking who "gives" them, how and why *is* a legitimate pursuit.[3]

3. The nature of tradeoffs

The notion of tradeoffs derives naturally from Friedman's distinction: there can be *no tradeoffs* in cases of a single criterion: one cannot "trade off" more for less or better for worse of the same thing. Consequently, *tradeoffs emerge* only in cases of multiple criteria.[4]

Here we emphasize that tradeoffs *emerge*: they are not fixed or natural properties of criteria, attributes or objectives. *Tradeoffs are imputed* by the set of *scarce means* (see Friedman 1962) and its properties. It would be erroneous to treat tradeoffs as if being the real properties of specific criteria, objectives or dimensions.

Whether or not there are tradeoffs depends not on alternative ends but only on scarce means. Although no single-criterion situation can have tradeoffs and therefore is not a subject of decision making, not all multiple-criteria cases will be characterized by tradeoffs: tradeoffs emerge or do not emerge on the basis of the means (feasible set

[3] It is appropriate to note that when resources are "given" *a priori*, their market prices become irrelevant (sunk costs). In all fairness, linear programming, as first proposed by Kantorovich in the USSR of the late 1930s, *was* appropriate for the times and place. However, why was LP, as a tool of Soviet central planning, adopted for the post-war "free-market" economies of the West, without considering market prices of resources, remains unexplained, if not inexplicable.

[4] Aggregating multiple criteria (or attributes) into a single super-function (like utility function) forms a single aggregate criterion and therefore does not pertain to human decision making, as no tradeoffs along the same function (regardless its complexity) are possible.

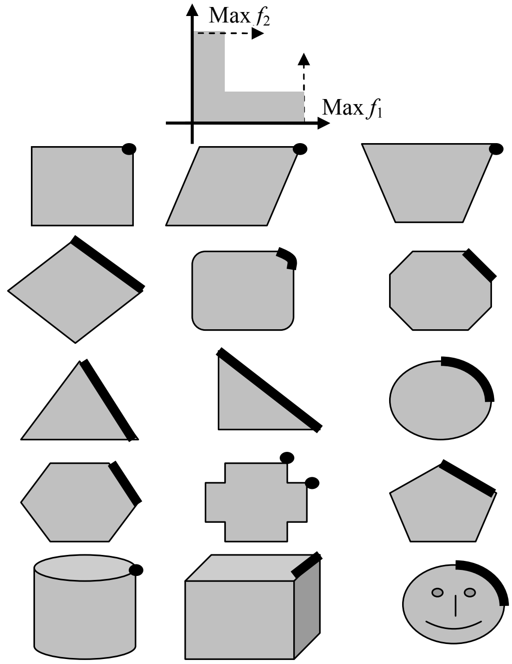
of alternatives) configuration. *Tradeoffs are the properties of the means, not of criteria or objectives*.

Yet, popular-science statements about criteria, like “there are tradeoffs between cost and quality”, are often accepted at their face value, as facts of reality.

What are criteria? Criteria are simply measures or *measuring tapes* for evaluating (measuring) objects of reality (things, alternatives, options, or strategies). There is a fundamental difference between measures and measured objects. Measuring “tapes” (length, volume, weight, sweetness, etc.) are quite different from apples, oranges and other measured alternatives.

There can be no tradeoffs between measures (or measuring tapes). Measures of cost and quality do not produce tradeoffs, the set of evaluated (measured) choices (alternatives, options) does. It is the configuration (size, shape and structure) of the feasible set (the measured “object” of alternatives, options and strategies) that is capable of producing or bringing forth any tradeoffs.

Figure 1. Optimality and Pareto-optimal solutions are the function of the feasible set, not of the criteria or objectives themselves



In Figure 1 there are two “conflicting” objectives, f_1 and f_2, both to be maximized over the changing array of feasible sets. The purpose here is to show that tradeoffs, conflicts, or any other forms of relationship between criteria or objectives, are *not* inner attributes of the measures, but are external attributes of the objects measured - in this case feasible sets, but also any sets of means, constraints, designs, etc.

It is also quite apparent, that the tradeoff boundary and its shapes, like the non-dominated set, Pareto-optimal solutions, efficiency frontier, productivity frontier, etc., are the properties of the set of options (objects of measurement), and not of the set of measures (criteria of measurements). This is significant because in order to truly maximize any objective function(s), one has to optimize the feasible set; the rest is a mere valuation and technical computation.

Observe that the identical pair of functions (criteria or evaluation measures) engenders tradeoff boundaries of different shapes and sizes, including the *no-tradeoffs cases*.

Because different configurations of means (different feasible sets) give rise to different solution configurations (different tradeoff or nondominated sets), the question of securing the best or optimal decision faces a new challenge: Any decision can undoubtedly be improved through changing the configuration of means (reshaping feasible sets of alternatives) while it clearly cannot be improved through re-computing over an *a priori* given and fixed set of alternatives. Consequently, *modern* decision analysis should be more about reshaping the means in order to attain a tradeoffs-free design as closely as possible, rather than struggling with unwanted tradeoffs brought forth by inadequate design of means.

Decision making is more about the scarce means (and the nature of their scarcity) than about the multiple ends.[5] It is more about the process (and its coordination) and less about its outcome (and its computation). An optimally coordinated and designed process will lead to an optimal outcome, but not *vice versa*. In fact, this conclusion is even stronger: a suboptimal process and poorly designed means *must* lead to inferior outcomes. A badly designed feasible set cannot be saved through mere computation, even if labeled "optimization".

Decision making therefore means *making it* through reconfiguration and design, not just *taking it* from a preconfigured and fixed set of means. Perhaps *decision design* (or *decision production*) would be more appropriate labels than conventional decision making.

The very notion of *a priori* feasibility is dubious in decision design because the purpose of means reconfiguration is to expand and redefine feasibility, not to accept it axiomatically. Innovation is not about doing the same thing better, but about doing things differently and, more importantly, doing different things. In decision design, it is not the efficiency (computation) but the effectiveness (design) that is of real consequence.

3.1 Pareto-efficiency

Modern welfare economics is based on the rather old idea of *Pareto-efficiency* (Pareto 1909), claiming that an allocation is efficient if it is impossible to move to another allocation which would improve some criteria and worsen no criterion. Based on this

[5] Even single ends can be improved through the reconfiguration of means, although no decision making is ever needed in single-end "decision making" because there are no tradeoffs. Mere computation (measurement and search) is necessary and sufficient.

assumption, a striking result can be obtained: in an economy of free markets, the resulting allocative equilibrium will be Pareto efficient.

How is it possible that economic "efficiency" is defined through tradeoffs? That is, one side, person or criterion can gain only if the other loses? How can they enter into a free-market transaction without both realizing a gain? Would anybody freely enter a transaction when one side must lose while the other gains? How can efficient allocation mean that scarce resources are being squandered (Zeleny 1981) through inefficiency?

The key is in a careful wording of the Pareto principle: it holds true, if and only if consumer tastes, resources, and technology are *given*. Of course, they never are. The production possibility frontier (see Figure 2) can be drawn only if the resources are assumed to be fixed and given. In the reality of free markets, individuals, firms and economies *continually* produce, purchase, destroy and sell resources, incessantly creating and re-creating the conditions where both sides of a transaction can benefit. Thus, resources are *never* given or fixed, but their optimal composition (or portfolio) is sought through entrepreneurial action. The existence of tradeoffs is the sign of inefficiency, not efficiency. This ancient failure of economics is that correct conclusions are often drawn from incorrect assumptions.

Because tradeoffs emerge only with multiple criteria, ends, objectives or dimensions, in the case of the utility function (which is a one-dimensional scalar) we have to shift to agents, i.e. persons holding and adhering to such utilities. The tradeoffs between agents (decision makers, consumers, providers, etc.) are pre-defined. At the Pareto-efficient point no free transaction is possible because the gain of one person is the loss to the other.

In a free market nobody would *freely* enter a transaction with the prospect of loss (except when speculating, betting, gambling or stealing). The basic premise of a free market is that *both* sides of a transaction must benefit – otherwise such transaction can take place only in an open (uninhibited) market, facilitated by manipulation, deception, exploitation, theft or just plain robbery. Tradeoffs-based transactions cannot be carried out freely by market participants.

Free markets are regulated in order to *protect* from harm the gains of *both* sides of a transaction. *Open markets* are unregulated (or poorly regulated), do not protect from harm and let the customer beware (*caveat emptor*). Free markets are tradeoffs-free; open markets are tradeoffs-based.

The difference between free and open markets is therefore palpable. The rules regulating free markets protect the participants from harm and assure that both sides of a transaction benefit. The under-regulated open markets allow manipulation and deceptions, producing benefits to one side at the cost to the other side (tradeoffs). Open markets are an easy target of governmental interference and intervention on behalf of one or the other side. While regulation protects both sides from harm, intervention tilts the market towards one side: it represents a violation of a free market and thus forms a breeding ground for political dominance over economics, opening space for nationalization, socialism, communism, dictatorship and assorted pseudo privatizations of the "tunneling" variety.

Regulation is a set of *rules* applied equally and fairly to *all* market participants in order to protect free markets. For example, no food manufacturer may use harmful additives. *Intervention* is an explicit *act* (usually by government) to create non-market advantage (disadvantage) to one participant at the cost to another participant. For example, one auto manufacturer will receive financial injection while the others shall not. The crisis of 2008 has clearly emerged at the intersection of too little regulation and too much intervention.[6]

3.2 A note on macro-tradeoffs

The most famous macro-tradeoff is the so called *Phillips curve* (Phillips 1958). Phillips came up with an empirical negative tradeoff between the rate of inflation and the level of unemployment. This innocuous curve was later "remade" into a policy tool by Samuelson and Solow (1960), in order to save the Keynesian system through trading higher inflation for lower average unemployment and *vice versa*.

Yet, in the long run, there is a natural rate of unemployment which could be combined with *any* level of inflation. The long-run Phillips curve is vertical; inflation is a monetary rather than a real phenomenon, as the stagflation of the 1970s confirmed. Later, Taylor (1979) concluded that there is no long-run tradeoff between the level of output and the level of inflation – but for their variability of fluctuations. Taylor's tradeoff has been derived from a policy choice, not from empirical observation. Mere weighting of central bank objectives (inflation and output targets) creates tradeoffs by definition; the policy itself "injects" tradeoffs into the economy where no empirically observable tradeoffs exist.

Yet, the objectives are just measures and no tradeoffs can ever exist between them *per se*, as observed in Figure 1.

Just by plotting differentially weighted points we generate an *efficiency frontier*, showing tradeoffs where none in fact exist. This tradeoff curve will be different for every assumed economic model of the economy. The *idea* of policy tradeoff has been imputed and become embedded in economic policy without any empirical support. Friedman took advantage of the "missing tradeoffs", denied two separate and independent objectives for the Fed and equally falsely proposed one and only one objective: to keep the price level steady. He claimed that what is involved is not a tradeoff but a direct cause-effect.

Friedman's diagnosis was correct but his cure was not. The lack of empirical tradeoffs among multiple objectives is not explained by assuming single objectives, but by recognizing that tradeoffs are not the properties of measures, but of the object they purport to measure.

One could similarly analyze all other models of traditional macroeconomics, well beyond the scope of this short discourse.

Let us consider, as an example, the traditional *productivity frontier*, comparing the delivered non-price buyer value and the relative cost position, as in Figure 2. The frontier describes the maximum value that a company can deliver at a given cost under

[6] The fashionable governmental oxymoron of sufficient regulation but poor enforcement or oversight cannot hold water. An unenforced or unenforceable regulation is bad or ineffective regulation.

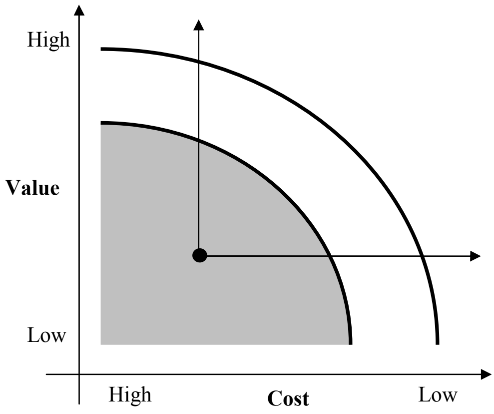
the best currently available circumstances. Observe that only companies operating below the productivity frontier are in a tradeoffs-free environment and can improve *both criteria* by moving towards the frontier. Once on the frontier, such companies can only trade off value against cost, by moving laterally along the frontier, back and forth.

3.3 Productivity frontier

Another key question of economics is whether the individual, firm or economy could produce more of some goods while producing no less of other goods? The answer is always yes and so the society is always wasting resources in production. This *productive efficiency* depends on how resources are selected, purchased, organized and coordinated, i.e. are they assembled and operated in an optimal manner? Resources are never given (except in centrally planned economies), but must be produced and purchased to form an optimal portfolio for any economic agent.

In Figure 2, even as the productivity frontier shifts outward (due to technological improvements and progress), the firms scramble again for a temporarily tradeoffs-free environment, only to see such "advantage" quickly dissipated as competitors copy each other and are forced to face the customer-unfriendly tradeoffs region.

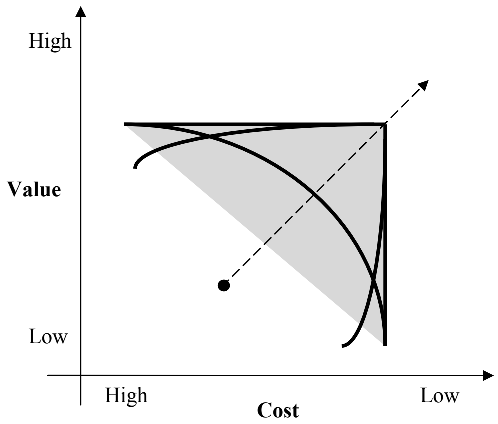
Figure 2. Tradeoffs-based improvement



The situation in Figure 2 is loaded with traditional assumptions. The tradeoffs between value and cost are assumed to exist *a priori*: only then can the frontier be drawn. No differentiation of means and goals is present; companies cannot design their own frontiers by engaging in different activities and different ways of carrying them out, etc. This is not how the real world works.

In Figure 3 we capture how companies redesign and reengineer their own processes and operations (reallocate their resources), so that the frontier (tradeoffs) is eliminated and the tradeoffs-free environment can be continually expanded and improved upon. The shaded area (the universe of corporate activities) of Figure 3 represents a distinct advantage and improvement over the shaded area of Figure 2. The situation in Figure 3 is a true, long-term strategic advantage, while the situation in Figure 2 requires continuous operational improvements and tradeoff choices, without fully satisfying the customer.

Figure 3. Tradeoffs-free improvement



Not only criteria, but the very purposes of decision making are clearly multiple. One should identify the best (optimal) solution through balancing multiple criteria. There is no single-criterion decision making, as there is nothing to balance and everything collapses into mere *measurement and search* computations. We have also established that decision making involves not only *a priori* fixed, given alternatives, but its most significant mode appears to be the design of the best (optimal) set of alternatives. If the decision making process is designed to search and configure the best possible set of alternatives, then the mere choice of the best decision is implied and can be explicated by computation.

There are several rules that have to be respected:

(i) What is determined or given (not just proposed) *a priori* cannot be subject to subsequent optimization and thus, clearly, does not need to be optimized: *it is given*.

(ii) What is not yet given must be selected, chosen or identified and is therefore, by definition, subject to optimization.

(iii) Consequently, different optimality concepts can be derived from distinctions between what is given and what is yet to be determined in problem solving, systems design or decision making.

Traditionally, by optimal solution or optimal decision making we implicitly understood maximizing (or minimizing) a single, pre-specified objective function (criterion) with respect to a given, fixed set of decision alternatives (situation constraints). Both the criterion and decision alternatives are given, only the (optimal) solution remains to be explicated (computed). A good example would be maximization of any aggregate function (like multi-attribute utility function and the like) with respect to predefined alternatives. That is not decision making but computation.

4. The Eight problems of optimization

There are at least *eight distinct optimization* problems, all mutually irreducible, all characterized by different applications, interpretations and mathematical/computational formalisms. They are displayed in Table 1.

Table 1. Eight problems of economic optimization

Given	Number of criteria	
	Single	Multiple
Criteria & alternatives	Traditional "optimality"	MCDM
Criteria only	Optimal design (De novo programming)	Optimal design (De novo programming)
Alternatives only	Optimal valuation (Limited equilibrium)	Optimal valuation (Limited equilibrium)
"Value complex" only	Cognitive equilibrium (Matching)	Cognitive equilibrium (Matching)

Observe that we use the simplest classification: single versus multiple criteria against the extent of the "given": ranging from "all-but" to "none except". The traditional economics, utility and multi-attribute utility theory, characterized by given alternatives and a single criterion, are displayed as the first cell of the first row. It naturally appears to be the most remote from optimal conditions or circumstances for decision making and optimization, as is represented by *cognitive equilibrium* (optimum) with multiple criteria (last cell of the last row). Current MCDM appears as the second cell of the first row, etc.

Elaborating the eight individual problems of economics is beyond the subject of this summary paper. An interested reader can consult relevant works of the author (Zeleny 1998, 2005).

Thus, to answer the often posed question, "*Are tradeoffs really necessary?*", the answer is no: *tradeoffs are not necessary*. In fact, their existence signals economic and decision inefficiency. Pursuing and achieving lower cost, higher quality and thus improved flexibility, all at the same time, is not only possible but clearly desirable and necessary in solving current economic problems.

Conventional wisdom recommends dealing with multiple criteria conflicts via "tough choices" and a "careful analysis" of the tradeoffs. Lean manufacturing has apparently eliminated the tradeoffs among productivity, investment and variety. "Quality and low cost" and "customization and low cost" were long assumed to be tradeoffs, but companies are now successfully overcoming such "wisdoms".

Needless to say that standard economics paradigm, the economic literature or multi-attribute utility theory simply demonstrate that trade-off evaluations and decisions are frequently painful and almost always tedious. These sources do not question their existence or contextual independence. Yet, tradeoffs are properties of incorrectly designed systems and thus can be eliminated by designing better, i.e. optimal systems.

5. Profit Maximization

The difference between *optimizing the given* (tradeoffs-based) system and *designing the optimal* (tradeoffs-free) system is of crucial importance in economics. It goes to the very core of free-market assumptions: profit or utility maximization by firms and individuals.

Indeed, it is impossible to direct firms solely by the goal of profit-maximization, with the now dissipated exception of the likes of Goldman Sachs. For most firms, there is this mildly annoying but necessary intermediating function of producing and developing goods and services that people want to buy and that are worth more in the market than they cost to produce. In other words, a firm does not *just* maximize profits, but must organize and coordinate its resources properly (optimally) so that it *can do* so (maximize profits).

Although the literature is rich about definitions and calculations of profit functions, it remains silent about *how* should profits be maximized. Should firms just do their best or second best? Should they "maximize" profits even while some resources are wasted? Should the resources be organized in the profit-maximizing fashion or will *any* given resource configuration do?

Then there is the problem of *ceteris paribus*: can we calculate the marginal product of a production factor while holding constant the input of all other factors? Is the assumption of "holding all factors constant except one" rational, as it is being promoted even in better textbooks (Begg et al. 1987)? The fact is that the factors of production are not independent and we cannot change one while holding all others unchanged. All factors of production are interdependent and can only be changed in synchrony, together as a system — as a *portfolio* of resources. (If a firm inputs an additional axle, it has to also get four more wheels and hire an extra driver as well as buy more gasoline, etc.) Factors of production form a matrix: all entries have to be adjusted as a whole, not *per partes*.

It should be clear that rational economic agents can maximize profits in *at least* two (see Table 1 for all eight options) fundamentally different and mutually exclusive modes:

(i) Manage (operate) a *given* system — so that a profit function is maximized.

(ii) Design an *optimal system* — so that its management (operation) leads to maximum profits.

This distinction is independent of the actual formula for profit definition or calculation. It is a fundamental distinction between a system given *a priori*, and a system designed *a posteriori*, i.e. after the process of optimization.

These two forms of profit maximization are clearly not identical. In the first case, one requires doing one's best and squeezing the maximum possible profits from a *given* system. This is known as profit maximization. In the second case, one designs (re-engineers) resources of a profit-maximizing system so that doing one's best leads to maximum profits. This is also profit maximization.

One cannot develop sound economics without defining its basic terms. Profit maximization, being so fundamental, should not mean two different things. The knowledge, skills and expertise required are entirely different in the two cases: to coordinate and manage the given is fundamentally different from coordinating and managing the design of the optimal.

Because the second case is, *ceteris paribus*, always superior to the first one, we are facing two strategically different concepts of profit maximization. It *does* matter — in business, economics or management — which particular mode of profit maximization the individuals, corporations or economic cultures prefer: free markets are bound to reward those who consistently adhere to *the second* mode of profit maximization — the optimal design of profit-maximizing systems — while punishing those who just struggle to do their best with their worst.

Let us now address the second row of Table 1.

6. Single-criterion De novo programming

The traditional resource allocation problem in economics is modeled via standard *linear programming* formulation of the single-objective product-mix problem as follows:

$$\max\ cx \text{ s.t. } Ax \leq b,\ x \geq 0 \tag{1}$$

That is, given the levels of m resources, $b = (b_1, \dots, b_m)$, determine the production levels, $x = (x_1, \dots, x_n)$, of n products, so as to maximize the value of the product mix $cx = \sum_j c_j x_j$. Because all components of b are determined *a priori*, problem (1) deals with the *optimization of a given system*.

When the purpose is to *design an optimal system*, the following formulation is of interest:

$$\max\ cx \text{ s.t. } Ax - \beta \leq 0,\ p\beta \leq B,\ x,\ \beta \geq 0 \tag{2}$$

That is, given the unit prices of m resources, $p = (p_1, \dots, p_m)$, and the total available budget B, allocate the budget so that the resulting *portfolio of resources* $\beta = (\beta_1, \dots, \beta_m)$ maximizes the value of the product mix. We assume that $A > 0$, $p > 0$ in (2).

It can be shown that the optimal design problem (2) is equivalent to a continuous "knapsack" problem (3) below:

$$\max\ cx \text{ s.t. } Cx \leq B,\ x \geq 0, \tag{3}$$

where $C = [C_1, \dots, C_n] = pA$. For the equivalency of solutions to problems (2) and (3), see Hessel and Zeleny (1987).

Since the "knapsack" solution is

$$x^* = \left[0, \dots, \frac{B}{C_k}, \dots, 0\right]^T, \tag{4}$$

where

$$\frac{c_k}{C_k} = \max_j \frac{c_j}{C_j}, \tag{5}$$

the optimal solution to (3) is given by (4) and

$$\beta^* = Ax^*. \tag{6}$$

Observe that under present assumptions this solution is nondegenerate and unique.

Duality. It can be shown that also the dual of (3) determines the solution of the dual of (2), as is shown in Hessel and Zeleny (1987) and only utilized here.

The dual of (3) is:

$$\min \ Bv \text{ s.t. } C^T v \geq c, v \geq 0, \tag{7}$$

so that its optimal solution, v^*, follows from (5):

$$v^* = \frac{c_k}{C_k} \tag{8}$$

Furthermore, (6) implies that $p\beta^* = B$; differentiating this equality with respect to β^* and using (8) gives:

$$v^* = [v_1^*, \ldots, v_m^*] = pv^* \tag{9}$$

as dual prices for the corresponding constraints of (2). Dual prices of resources are proportional to their costs, so that contributions of all resources in real terms are equal.

We can now formulate some characteristics of optimally designed product-mix problems:

(i) The optimal design leads to *full-capacity utilization of all resources*. Any underutilized resources or excessive resource capacities are the sign of inefficient allocation and suboptimal performance, as in traditional linear programming (2).

(ii) Dual (shadow) prices of resources are proportional to their costs, so that contributions of all resources in real terms are equal. Rewriting (9) as

$$\frac{v_i^*}{v_j^*} = \frac{p_i}{p_j} = v^* \tag{10}$$

yields the well-known result on the marginal rate of input substitution and confirms the optimality of resource allocation.

(iii) The value of no resource can exceed its value in the optimally designed system (2).

(iv) In real terms, no resource can be valued above any other. The essence of optimal design lies in a *perfect balance* among its resource components.

(v) No reimbursement for an additional unit of a resource can exceed the actual contribution of this unit to the objective function.

(vi) Let additional units of resource i, Δb_i, involve additional budget expense of $\Delta B = p_i \Delta b_i$. Then the optimal allocation of B between various resources must satisfy $\Delta\beta^* = A\Delta x^*$, or

$$\frac{\Delta\beta_j^*}{\Delta\beta_i^*} = \frac{a_{jk}}{a_{ik}} \tag{11}$$

(vii) Any deviations from condition (11) imply less-than-efficient allocation.

Numerical Example. A simple numerical demonstration of the optimal-design procedure is seen in the following LP problem:

$$\begin{aligned} \max\ & 400x_1 + 300x_2 \\ \text{s.t.}\quad & 4x_1 \le b_1 \\ & 2x_1 + 6x_2 \le b_2 \\ & 12x_1 + 4x_2 \le b_3 \\ & 3x_2 \le b_4 \\ & 4x_1 + 4x_2 \le b_5 \end{aligned}$$

where $p_1 = 30$, $p_2 = 40$, $p_3 = 9.5$, $p_4 = 20$ and $p_5 = 10$ are market prices ($ per unit) of the resources b_1 through b_5 respectively. Let also $B = 2600$ and $x_1 \le 6$ (e.g., maximum demand limitation). Two basic approaches can be used to determine b_1 through b_5 optimally:

(i) *Accounting approach*. In an optimally designed system, all of the constraint inequalities must become equalities. Then,

$$30b_1 + 40b_2 + 9.5b_3 + 20b_4 + 10b_5 = 2600.$$

Substituting the left-hand sides of constraints, we get:

$30(4x_1) + 40(2x_1 + 6x_2) + 9.5(12x_1 + 4x_2) + 20(3x_2) + 10(4x_1 + 4x_2) = 2600$, which is reduced to $354x_1 + 378x_2 = 2600$ and thus $x_1 = 7.35 - 1.07x_2$. Maximize $400x_1 + 300x_2 = 400(7.35 - 1.07x_2) = 2940 - 426x_2$ by making $x_2 = 0$. So, $x_1 = 7.35$, $x_2 = 0$ and $b_1 = 29.4$, $b_2 = 14.7$, $b_3 = 88$, $b_4 = 0$, $b_5 = 29.4$.

If x_1 cannot exceed 6, then $x_1 = 6$ and $x_2 = (7.35 - 6)/1.07 = 1.26$ and thus $b_1 = 24$, $b_2 = 19.56$, $b_3 = 77.04$, $b_4 = 3.78$, $b_5 = 29.04$.

(ii) *De novo approach*. Solve the simple LP-knapsack problem:

$$\begin{aligned} \max\ & 400x_1 + 300x_2 \\ \text{s.t. } & 354x_1 + 378x_2 \le 2600 \end{aligned}$$

By choosing the largest of the two ratios 400/354 and 300/378 and making the corresponding variable (here x_1) as large as possible: $x_1 = 2600/354 = 7.35$. Because $x_1 = 6$, then $x_2 = 2600 - 354(6) = 476/378 = 1.26$; the same result as above is obtained.

Note. We use "fixed" budget B only as a computational tool in order to establish initial levels of resources in optimal design. Depending on the outcome, further increases or decreases in B can be quickly stipulated, following the optimal pattern. That is, also budget B can (and should) be optimized in a market-priced system. This will become clearer from *optimality-path ratios* in Section 7.

Traditional linear programming is of little interest in non-socialist economics because it addresses a simple technological rather than economic problem (recall Friedman) as it "optimizes" an inherently suboptimal system, which is fixed and given *a priori*, with *no market prices* of resources, and a single objective function where no tradeoffs are possible. Its "optimization" is just a simple computation, its solution being fully determined by the problem's structure. No optimization (in the sense of system improvement) takes place; the system remains unchanged by the calculation, remaining suboptimal. No optimal design of a system takes place.

In the next section we summarize *De novo programming*, which designs the optimal portfolio of resources in dependency on market prices and an investment budget, and with respect to multiple objective functions, redesigns the shape of the feasible set so that the tradeoffs are fully eliminated. In economic multiobjective problems, the existence of tradeoffs is always a sure sign of suboptimality, poor system performance and consumer dissatisfaction.

7. Multiple criteria De novo programming

Let us formulate a linear programming problem (also Zeleny 1990):

$$\max\ Z = Cx \text{ s.t. } Ax - b \leq 0,\ pb \leq B,\ x \geq 0, \tag{12}$$

where $C \in \Re^{q\times n}$ and $A \in \Re^{m\times n}$ are matrices of dimensions $q \times n$ and $m \times n$, respectively, $b \in \Re^m$ is the m-dimensional unknown vector of resources, $x \in \Re^n$ is the n-dimensional vector of decision variables, $p \in \Re^m$ is the vector of the unit prices of m resources, and B is the given total available budget.

Solving problem (12) means finding the optimal allocation of B so that the corresponding resource portfolio b maximizes simultaneously the values $Z = Cx$ of the product mix x.

Obviously, we can transform problem (12) into:

$$\max\ Z = Cx \text{ s.t. } Vx \leq B,\ x \geq 0, \tag{13}$$

where $Z = (z_1,\dots,z_q) \in \Re^q$ and $V = (V_1,\dots,V_n) = pA \in \Re^n$.

Let $z_{k^*} = \max\ z_k$, $k = 1,\dots,q$, be the optimal value for the k-th objective of problem (13) subject to $Vx \leq B$, $x \geq 0$. Let $Z^* = (z_{1^*},\dots,z_{q^*})$ be the q-objective value for the ideal system with respect to B. Then, a metaoptimum problem can be constructed as follows:

$$\min\ Vx \text{ s.t. } Cx \geq Z^*,\ x \geq 0. \tag{14}$$

Solving Problem (14) yields x^*, $B^*(=Vx^*)$ and $b^*(=Ax^*)$. The value B^* identifies the minimum budget to achieve Z^* through x^* and b^*.

Since $B^* \geq B$, the *optimum-path ratio* for achieving the ideal performance Z^* for a given budget level B is defined as:

$$r^* = \frac{B}{B^*} \tag{15}$$

We establish the optimal system design as $(\mathbf{x}, \mathbf{b}, \mathbf{Z})$, where $\mathbf{x} = r^* x^*$, $\mathbf{b} = r^* b^*$ and $\mathbf{Z} = r^* Z^*$. The optimum-path ratio r^* provides an effective and fast tool for the efficient optimal redesign of large-scale linear systems.

There are two additional types of budgets (other than B and B^*). One is B_j^k, the budget level for producing the optimal x_j^k with respect to the k-th objective, referring back to the single-objective De Novo programming problem.

The other, B^{**}, refers to the case $q \leq n$ (the number of objectives is less than the number of variables). If x^{**} is the degenerate optimal solution, then $B^{**} = Vx^{**}$ (see Shi 1995). It can be shown that $B^{**} \geq B^* \geq B \geq B_j^k$, for $k = 1, \ldots, q$.

Shi (1995) introduces six types of optimum-path ratios:

$$r_1 = \frac{B^*}{B^{**}},\ r_2 = \frac{B}{B^{**}},\ r_3 = \sum \lambda_k \frac{B_j^k}{B^{**}},\ r_4 = r^* = \frac{B}{B^*},\ r_5 = \sum \lambda_k \frac{B_j^k}{B^*},\ r_6 = \sum \lambda_k \frac{B_j^k}{B}$$

They lead to six different policy considerations and optimal system designs. Comparative economic interpretations of all optimum-path ratios are dependent on the decision maker's *value complex* (Zeleny 1998).

Numerical example. The following numerical example is adapted from Zeleny (1984, 1986):

$$\begin{aligned}
\max \quad & z_1 = 50x_1 + 100x_2 + 17.5x_3 \\
& z_2 = 92x_1 + 75x_2 + 50x_3 \\
& z_3 = 25x_1 + 100x_2 + 75x_3
\end{aligned}$$

$$\begin{aligned}
\text{s.t.} \quad 12x_1 + 17x_2 &\leq b_1 \\
3x_1 + 9x_2 + 8x_3 &\leq b_2 \\
10x_1 + 13x_2 + 15x_3 &\leq b_3 \\
6x_1 + 16x_3 &\leq b_4 \\
12x_2 + 7x_3 &\leq b_5 \\
9.5x_1 + 9.5x_2 + 4x_3 &\leq b_6
\end{aligned} \tag{16}$$

We assume, for simplicity, that the objective functions z_1, z_2, and z_3 are equally important. We are to identify the optimal resource levels of b_1 through b_6 when the current unit prices of resources are $p_1 = 0.75$, $p_2 = 0.60$, $p_3 = 0.35$, $p_4 = 0.50$, $p_5 = 1.15$ and $p_6 = 0.65$. The initial budget $B = \$4658.75$.

We calculate $Z^* = (10916.813, 18257.933, 12174.433)$ with respect to the given $B(\$4658.75)$. The feasibility of Z^* can only be assured by the *metaoptimum* solution $x^* = (131.341, 29.683, 78.976)$ at the cost of $B^* = \$6616.5631$.

Because the optimal-path ratio $r^* = 4658.75/6616.5631 = 70.41$, the resulting $\mathbf{x} = (92.48, 20.90, 55.61)$ and $\mathbf{Z} = (7686.87, 12855.89, 8572.40)$. It follows that the optimal portfolio **b**, with respect to $B = \$4658.75$, can be calculated by substituting **x** into the constraints (16). We obtain the optimal portfolio of resources as:

$$
\begin{aligned}
b_1 &= 1465.06\\
b_2 &= 910.42\\
b_3 &= 2030.65\\
b_4 &= 1444.64\\
b_5 &= 640.07\\
b_6 &= 1299.55
\end{aligned} \tag{17}
$$

If we spend precisely $B = \$4658.8825$ (approx. $\$4658.75)$ we can purchase the optimum portfolio of resources (17) at current market prices, allowing us to produce **x** and realize **Z** in criteria performance. No better solution can be realized for given amount of money.

8. Added value

One of the most common examples of agent tradeoffs is the win-lose tension between the producer and the consumer. Because the *price paid* is still a main economic category, its increase brings higher added value (and profits) to the producer at the cost of a lowered added value to the consumer. The lowering of the price paid has the opposite effect. So there are constant tensions between producers and consumers in trading off total added value between themselves, rather than increasing its apportioned levels for both at the same time,[7] as would free-market transactions require: both parties to a transaction must benefit in order to enter into the transaction freely, i.e., without coercion or deception.

Decision making is a process and its coordination of action stages or phases, as well as their sequencing and structural configuration, is a matter of knowledge and skills. It is this *knowledge* (purposeful coordination of action) that adds value to the means or inputs. One of the dominant inputs is *information*. That is why a clear distinction between information (input) and knowledge (process) is so crucial in economics. Knowledge, according to Hayek (1937, 1945), is the main concept in free-market economics.[8] While knowledge is the demonstrated capacity of coordinating action, information is its symbolic description, a digitizable record of past, present and future actions. As such, information *per se* is of limited value (rapidly becoming a commodity, often accessible for free): it acquires value only through being internalized in the decision-making process and transformed into knowledge, i.e. only through action and the value of its outcome (Zeleny 2005).

[7] This ancient struggle over the price paid is exemplified by the Turkish bazaar and its vigorous haggling over price as a flexible tool for dividing fixed added value between competing agents. In more advanced economies, prices paid become relatively fixed and the potential for increasing added value for both sides is thus created.

[8] We could add *trust* as the second most important concept, as current crisis has demonstrated in abundance.

All this is rather straightforward as "uninternalized" information becomes just a background clutter, white noise or information overload. Information is thus being transformed, at an increasing rate, into a sort of *exformation*, or informational waste. While information (and exformation) becomes plentiful, human attention span is becoming a scarce resource.

In contrast, there can never be any "knowledge overload" and so the only effective and safe way to improve performance and decision making is through knowledge, i.e. through a purposeful coordination of production and decision-making processes.

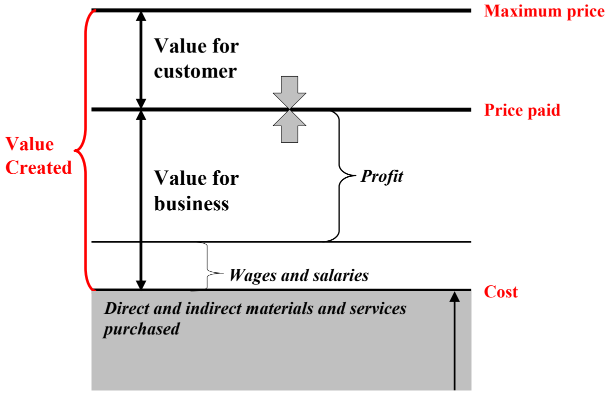
8.1 What is Added Value?

Knowledge is measured by the value created by coordination of effort. Action and processes add to materials, technology, energy, services, information, time and other inputs used or consumed. *Knowledge is measured by added value*.

In any business (and human) transaction, value has to be *added to both* participants (or sides): to the provider *and* to the customer. Adding value to both is what makes the transaction satisfactory and sustainable: it creates a free market.

There are two kinds of value to be created: *value for the business* and *value for the customer*. Both parties must benefit: the business — in order to produce it; the customer — in order to purchase it. In the global age it is precisely this business-customer *value competition* that is emerging as the hardest and the busiest battleground.

Figure 4. Adding Value for the Customer and Business



In Figure 4 we explain the process of creating new value. This is crucial for measuring knowledge and for the identification and assessment of innovation.

First, the customer pays for the service or product: the *price paid*. The producer subtracts the *cost incurred*, including all direct and indirect materials and services purchased. The difference is the *added value* for the firm (producer, provider). This added value can also be interpreted as the *value of knowledge* engaged in producing the service or product. In order to pay wages and salaries, the production process and its coordination must generate added value. Added value is the only source of wages, salaries *and profits*.

If the added value does not *cover* the wages and salaries, then they must be correspondingly lowered. If no value has been added, then the value of knowledge is zero and no payment can be attributed to it. The business must add enough value in order to *cover* at least the salaries and wages of its workers and managers. If even more value has been created, then *profits* can be realized, up to the price paid.

The customer, of course, must be willing and ready to pay more for the service/product than he actually paid. The *maximum price* the customer would be willing to pay must exceed the price the producer has asked for. The difference is the added *value for the customer*.

If there is no value for the customer — the maximum price is lower than the price to be paid — then the customer would not purchase the service or product. In a competitive market, the customer pays money only for the value received, i.e. the value for the customer.

The proper entrepreneurial and long-term strategy must eliminate the tradeoff between the firm and its customer. Such strategy concentrates on increasing the maximum price while lowering the costs — *both* at the same time, in a tradeoffs-free fashion. Both sides are then bound together in a mutually beneficial process of producing maximum added value. The price paid can remain relatively constant as long as both sides' benefits are increasing, avoiding the tradeoffs-based environment and decision making. Periodically, the optimal system is redesigned so that the portfolio of resources and its coordination maintains tradeoffs-free added value for both (maximum profits are implied), as the example in Section 7.

9. Conclusions

The areas of economics where tradeoffs-free thinking and economic calculus of De novo optimization can be applied are numerous. We cannot elaborate on all aspects of tradeoffs-free economics in this paper, but point out a few such areas with some references.

For example, the efficiency frontier of *portfolio analysis* (Markowitz 1952, 1959) is based on the assumption of tradeoffs between expected value and standard deviation of uncertain returns — i.e. tradeoffs between two moments of the same probability distribution of returns. Lerner (1971) has criticized variance-based measures of risk while Colson and Zeleny (1979) proposed a three-dimensional measure of risk. Problems of *conflict resolution* have often been mismanaged through preserving the tradeoffs (the win-lose conditions) which are the very cause of conflict. Yet, the tradeoffs-free *conflict dissolution* methodology has been proposed (see e.g. Zeleny 2005).

Tradeoffs are often assumed between key *policy objectives*. The *Phillips curve* postulates tradeoffs between unemployment and inflation (see Section 3.2), but similar arguments can be advanced for growth and inflation or growth and the balance of payments. Such tradeoff assumptions severely limit policy options even though arguments can be made for tradeoffs-free alternatives in all such cases. Also, quantitative economic policy based on game theory provides a useful application. The field of *games with multiple* (or vector) *payoffs* is only referenced here (Zeleny 1976) due to the lack of space. Traditional, scalar-payoffs based games are no longer sufficient for economics. Similarly, *environmental regulation* would benefit from a dynamic version of De novo programming and its optimal-path ratios.

Another area of tradeoffs is the false notion that through environmental *regulation* we have traded off a considerable portion of our national wealth for costly environmental protection. This can only be true if the cost of using harmful energy sources is very low. Once the price of such sources becomes sufficiently high, the growth and environmental protection go hand in hand. Not all regulation is harmful to growth: much regulation encourages innovation, increases the supply of public goods, and stimulates the development of new businesses. It is the abundance of (governmental) market *intervention*, not regulation, we should be concerned about.

Tradeoffs are the properties of available options (feasible sets), not of criteria, goals or objectives (measures). This has been the main message of this paper.

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