Czech Economic Review
Acta Universitatis Carolinae
Oeconomica

vol. 1 *September 2007* *no. 3, pp. 255–286*

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CREDIT RISK AND THE FINNISH ECONOMY

Abstract
The significance of credit risk models has increased with the introduction of the New Basel Accord, known as Basel II. The aim of this study is to examine default rate modeling. This paper follows two possible approaches to macro credit risk modeling, empirical models and a latent factor model based on Merton. We employ data over the time period from 1988 to 2003 for the Finnish economy, including time series of bankruptcy, numbers of firms and industry-specific data. Linear vector autoregressive models are used in the case of a dynamic empirical model. We examine how significant macroeconomic indicators determine the default rate in the whole economy and in industry-specific sectors. Since these models cannot provide microeconomic foundations, we employ a model with one latent factor, although multi-factor models are also considered. This estimation helps us to understand the relationships between credit risk and macroeconomic indicators. Both models can be used for default rate prediction or stress testing by central authorities.

Keywords: banking, credit risk, latent factor model, default rate

JEL classification: G21, G28, G33

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Acknowledgments: This paper originated from research done at the Bank of Finland Research Department. I would like to thank Alistair Milne, Esa Jokivuolle, Karlo Kauko, Tuomas Takalo, Jan Toporowski, Juko Vilmunen, Matti Virén, and other colleagues at the Bank of Finland for useful comments. All remaining errors are the sole responsibility of the author. I also wish to thank Elina Rainio for research assistance. Financial support from the IES (Institutional Research Framework 2005-2010, MSM0021620841) is gratefully acknowledged. The findings, interpretations and conclusions expressed in this paper are entirely those of the author and do not represent the views of either of the above institutions.

1. Introduction

Credit risk is one of the most important areas of risk management. Research into credit risk has rapidly expanded over the last decade. Credit risk plays an important role mainly for bank institutions. They are trying to develop their own credit risk models in order to increase their profits. A new wave of interest originated with the introduction of the New Basel Accord, known as Basel II.

Three approaches can be distinguished. The first approach – involving traditional models – is based on comparing client-specific information. The objective of these models is to provide a good prediction of future client quality. The default probability is obtained from empirical information. These models are widely used for business clients and this approach is also very popular for transition economies with insufficient capital markets. Models based on option pricing (structural models) are the second possible approach. They are based on financial pricing theory. Here, the value of a firm is modeled as an option price. Firm default is specified in relation to firm value and leverage. The third approach is summarized in so-called reduced form models. These models use the market bond price as the input, and from this information they try to derive the default probability and recovery rate. The aim of all the approaches is to provide an estimation of firm default probability and loss given default. Together with an estimation of exposure at default and effective maturity these credit risk components can be used to determine the capital requirement – the Internal Ratings-Based Approach (IRB).

One question which has become important is the relationship between credit risk models and the business cycle. Research on this relationship has increased during the last few years in particular. The targets of these studies are credit risk models taking into account the macroeconomic environment. Some papers are focused on developing a macro model for credit risk estimation. In general, these types of models try to estimate the default rate from macro data. These models are used for stress testing. This testing is emphasized by the New Basel Accord. Banks with IRB models must use stress testing in the assessment of capital adequacy. Stress testing must involve identifying possible events or future changes in economic conditions that could have a negative effect on the banks capital requirements (Basel Committee on Banking Supervision 2004). Macro models are also a very useful tool for central banks for research and management of banking system financial stability. Through the application of these models a central bank can estimate the impact of changing monetary policy or expected or unexpected macroeconomic shocks.

Two basic approaches to default probability modeling can be distinguished. Banks can base borrower assessments on the current economic conditions. Default probability is then made conditional on the point in the cycle. When risk assessments take into account possible changes in the macroeconomic climate, forward-looking ratings can then be derived. The second approach is important due to the possibility of implementing different types of cyclical policy. Macroeconomic models can help us to understand the influence of macroeconomic changes on default events.

This paper contributes to the contemporary research by comparing two basic approaches to macroeconomic default prediction. First, empirical models are introduced. Second, latent factor models based on Merton's idea (Merton 1974) are investigated. Our study is linked to previous research done at the Bank of Finland (Virolainen 2004). It extends the previous analysis of Finnish default data by introducing latent systematic risk factors. We try to offer an alternative to the previous study, where an empirical approach to modeling was employed. However, very similar macroeconomic indicators are used. Factor models can be a better way of modeling default rates, because they provide microeconomic foundations.

In this paper, we focus on developing macro models for default rate prediction. This paper sets out to investigate possible approaches to default rate macro modeling described in the literature, and to select a model for the Finnish economy. There are several reasons for being interested in the relationship between business cycle fluctuations and default. First, financial regulators need to have a good understanding of the potential downside credit risk in loan and corporate bond portfolios. They therefore need to be able to estimate the potential cyclical variability of default rates. Second, management and regulators will want to have some idea of the likely rate of default in the immediate future. Macroeconomic indices are informative indicators of future default rates, requiring direct modeling of these relationships. Third, as encouraged by the Basel Committee, banks need to be able to develop stress tests of their portfolio performance in business cycle downturns, and these tests should be interpretable in terms of the magnitude of some underlying macroeconomic shock.

This study can help in all these tasks. The latent factor model is a natural and popular way of estimating potential downside credit risks. This is why the latent factor model forms the basis of Pillar 1 of the New Basel Accord (Gordy 2003). But relatively little work has been done on estimating the crucial parameter, namely, the correlation with the systematic factor. Combining a latent factor model with macroeconomic indicators provides a natural test of the specification of the macro-relationship. If macro indicators are indeed informative predictors, then the share of fluctuations explained by the latent factor will be relatively small. The latent factor represents the unexplained component of the macro-model. We found that the latent factor remains important even with the inclusion of macro indicators. Therefore, both simulation and forecasting should allow for the latent factor as well as the observed macroeconomic indicators

This paper is structured as follows. Section 1 introduces related studies. Section 2 contains all the data considered in this study. Bankruptcy data as a proxy for defaults and macroeconomic indicators are described. Section 3 presents the macroeconomic credit risk models used. The dynamic models are discussed within the framework of empirical models. Linear dynamic vector autoregressive models and their vector error correction forms are used to investigate the relationship between the default rate and some macroeconomic indicators. Lastly, a more sophisticated nonlinear one-factor model is used for default rate modeling. This model is derived from the idea of modeling asset returns using systematic factors and idiosyncratic shocks. A multi-factor model is also suggested, but due to the complicated numerical solution, only one-factor models are estimated for the Finnish economy. Section 4 describes the results of the latent factor model for the Finnish economy. All relationships are investigated for the aggregate economy and also for five sector-specific industries (agriculture, manufacturing, construction, trade, transport). The last section concludes and discusses possible further research issues.

2. Related studies

Some studies focus on the effects of the business cycle on portfolio credit risk; others research the procyclicality of credit risk measurement or the relationships between financial crises and credit risk models. Four basic components are defined in the New Basel Accord under the Internal Ratings-Based Approach (Basel Committee on Banking Supervision 2004). These are: default probability, loss given default, exposure at default and effective maturity. In discussions about the relationship between the business cycle and credit risk models, the most important are default probability and loss given default. Some papers address the problem of the correlation between default probability and loss given default. In general, the default probability changes over time depending on the macroeconomic environment. Some models use a constant value of loss given default, but this also changes over time in practice. Many studies demonstrate this fact. The basic issue of the relationship between credit risk models and the economic cycle is the estimation of default probability as a function depending on time. Default probability is usually modeled by the default rate. This indicator is defined as the ratio between loans in default and total loans granted. This type of data at the aggregate level of the economy is sometimes very difficult to get hold of. In this case some approximation must be used. These models use aggregate variables to explain the default rate. Macro indicators are very often taken into account. Such models are able to model the impact of a macroeconomic shock on the credit industry.

This paper is related to the literature on the influence of the macroeconomic environment on credit risk models. Few papers focus on the issue of the relationship between the economic cycle and credit risk. Those that do can be divided into two groups. The first group uses company-specific information and tries to research the influence of the macroeconomic environment on individual risks. Other studies use aggregate data only and investigate the default rate in relation to macroeconomic indicators. This paper uses aggregate information only and therefore falls in the second group of papers.

In the context of the New Basel Accord, there are studies that investigate cyclical effects in credit risk models. They try to model the influence of cyclical policy on the bank capital requirement. This issue is dealt with in (Catarineu-Rabell, Jackson, Tsomocos 2003). They discuss the influence of different rating systems on a banks capital requirement. They conclude that when banks assess a borrower's probability of default, the assessment can be based on the current economic conditions or can take into account the effect on the borrower of possible adverse changes in the economic climate. They show that even this approach could lead to a 15% increase in bank capital requirements in recessions. Their results indicate that banks will not choose a stable approach. Given complete freedom banks will choose a countercyclical approach, reducing ratings in a recession, and if regulators prevent this, banks will adopt a procyclical approach.

Lowe (2002) examined whether credit risk is low or high in economic booms. He described how macroeconomic considerations are incorporated into credit risk models and the risk measurement approach that underlies the New Basel Capital Accord. Finally, he researched the influence of these measurement approaches on the macroeconomy. A survey of the literature on cyclical effects on default probability, loss given default and exposure at default can be found in (Allen, Saunders 2003). They noticed that although systematic risk factors have been incorporated into both academic and proprietary models for default probability, the same is not true for loss given default and exposure at default. Moreover systematic correlation effects between default probability and loss given default, default probability and exposure at default, and loss given default and exposure at default have been ignored in the literature.

Some studies have used latent factor models to investigate the effects of the business cycle on portfolio credit risk. These models are based on the Merton model. Cipollini, Missaglia (2005) attempt to integrate market risk with credit risk. An estimation and identification of a common shock underlying the business cycle was obtained by fitting a dynamic factor model to a large number of macroeconomic credit drivers. They noticed a relationship between default probability and recovery. Their empirical results suggest that ignoring the main feature of recoveries, as stochastic and dependent on default, can imply serious under provision of minimum capital requirements. Rösch (2003) estimated a one-factor model for the German economy. He used data on bankruptcies to estimate default probabilities and correlations between firm normalized asset returns. This model is estimated for the whole German economy and also for 16 industry-specific sectors. The one-factor model is also employed in (Rösch 2005). Two rating philosophies are distinguished: through the cycle versus point in time ratings. Data from Standard & Poor's were used. It was shown that point in time ratings will exhibit much lower correlations derived from the nonlinear one-factor model, and, thus, default probability forecasts should be more precise. As a consequence, the value-at-risk quantiles of the default distribution should be lower than those generated by through the cycle ratings. This fact may affect bank punishment in times of economic stress if the implied reduction of asset correlation is not taken into account when using point in time ratings.

Hamerle, Liebig, Scheule (2004) also used a static factor model, but they consider the effect of different assumptions about the error distribution function. The empirical analysis was based on a large data set of German firms provided by Deutsche Bundesbank. They used the logistic distribution function, in contrast to (Rösch 2003) and (Rösch 2005), where the normal distribution function is used. They found that the inclusion of variables which are correlated with the business cycle improves the forecasts of default probabilities. Céspedes, Martín (2002) studied a two-factor model for credit risk. They compared this model with the one-factor model employed in Basel II.

Lucas, Klaassen (2003) used simple mapping to cast discrete state regime switching models for credit risk into a continuous state factor model structure. They studied the implied default probabilities and asset correlations of the regime switching approach. They found that the correlations implied by the model are low, and may appear too low given typical estimates of asset correlations in the literature. They showed that asset and default correlations appear to be higher in recessions than expansions. Tasche (2005) investigated multi-factor extensions of the asymptotic single risk factor model and derived exact formulae for the risk contributions to value-at-risk and expected shortfall. He introduced a new concept of a diversification index as an application of the risk contribution formulae. He illustrated this concept with an example calculated with a two-factor model. The results indicated that there can be a substantial reduction of risk contributions by diversification effects. A three-factor structural model is developed for example in (Hui, Lo, Huang 2003).

Pesaran, Schuermann (2003) used the idea of a simple Merton-type credit model to model credit risk as a function of correlated equity returns of obligor companies. These equities are linked to correlated macroeconomic variables using an approach similar to the Arbitrage Pricing Theory. They estimated a global macroeconomic model for generating a conditional loss distribution using stochastic simulation. They analyze the impact of a shock to a set of specific macroeconomic variables on that loss distribution. Koopman, Lucas (2004) used a multivariate unobserved components framework to separate credit and business cycles. They used this model to describe the dynamic behavior of credit risk factors in relation to the real economy. They used data on real GDP, credit spreads, and business failure rates for the US economy. They distinguished two types of cycles in the data, corresponding to periods of around 6 and 11–16 years, respectively. Cyclical co-movements between GDP and business failures mainly arise at the longer frequency. They empirically showed a positive relationship between spreads and business failure rates and a negative relationship between spreads and GDP.

Some papers try to develop simple macroeconomic models of default rate predictions. These empirical models are derived from traditional models used for predicting individual risks. Few papers focus on the developing macroeconomic model of default rates. Virolainen (2004) estimated this kind of model for the Finnish economy. He used this model for stress testing and tried to investigate the influence of shocks on the expected and unexpected losses. His model is based on logistic regression. Pesola (2001) published a study of the role of macroeconomic shocks in banking crises. This study also used data on the Finnish economy.

3. Data description

We used monthly data on the Finnish economy for all calculations. Bankruptcy data and some macroeconomic indicators were employed.

3.1 Bankruptcy data

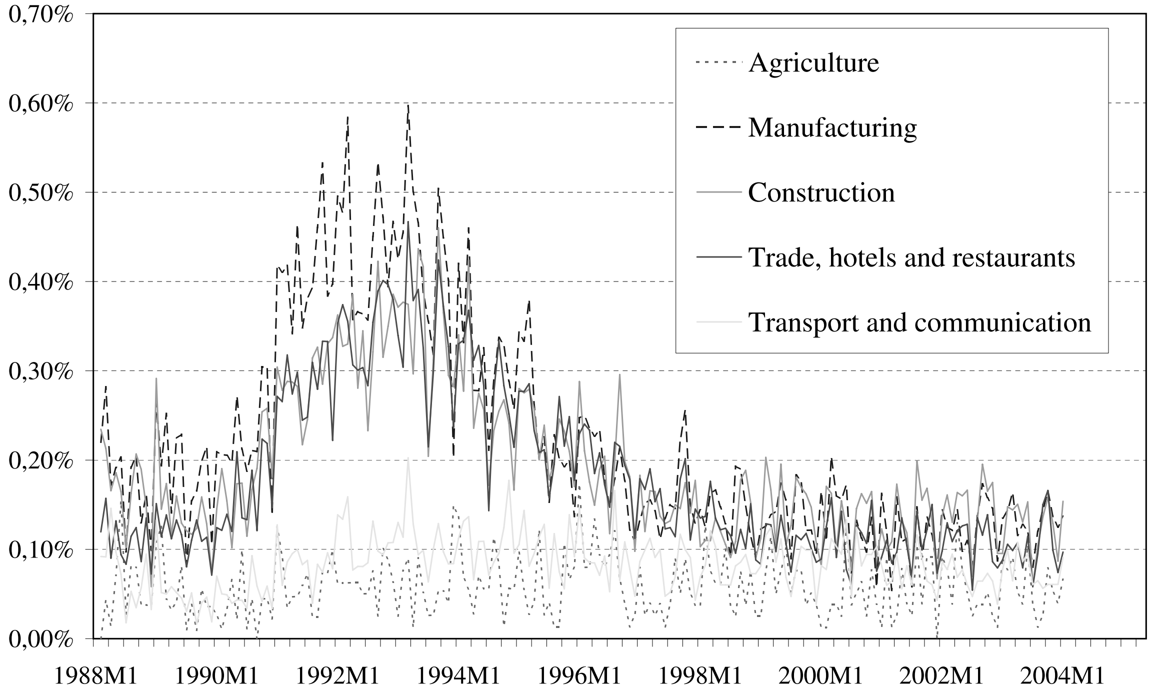
The numbers of companies in default were the most important time series in our analysis. Default was defined in the same way as in (Virolainen 2004). A defined default takes place when bankruptcy proceedings are instituted against a firm for the first time. We considered this definition to be more strict than the one commonly applied, but it is still a good approximation, and data on bankruptcies are available for the Finnish economy. A default event is commonly defined as payment delinquency in some minimum amount. The 12-month default probability is usually employed in credit risk assessments. Generally, an M-month default at time t is defined when a default event happens in time interval $(t, t+M]$ and the entity is not in default at time $t-1$. The given definition corresponds to a new default event. This indicator is monitored by financial institutions as well as central authorities. In this paper, all the calculations are based on monthly data. Monthly time series of company bankruptcies were available from 1/1988 to 5/2005. Time series of numbers of firms are available on a yearly basis from 1988 to 2003. The data on numbers of firms were disaggregated from the annual data.[1] We computed 1M-default rates as a ratio of the number of bankruptcies at time t and the number of firms at time $t-1$. As a result of this calculation, a time series of observed default rate approximations from 2/1988 to 1/2004 was available.

Figure 1 shows the 1M observed default rates in the Finnish economy. We computed industry-specific default rates as well as aggregate default rates for the whole economy. Data on active company numbers and bankruptcies were available for the following five industries: agriculture (AGR), manufacturing (MAN), construction (CON), trade, hotels and restaurants (TRD), and transport and communication (TRN), together with aggregate data for the whole economy. The same segmentation as in (Virolainen 2004) was used in this paper. The industry-specific default rates seem to converge at the end of the observed data, but there is significant differentiation during recessions. Increasing default rates during recessions were significant for MAN, CON, and TRD. The default rates for AGR and TRN were not significantly changed during recessions. One problem with the observed default rates data was a change in the bankruptcy law implemented in 1993.[2]

[1] The numbers of firms were disaggregated from the annual data using EKTA (Bank of Finland software).

[2] The law was changed to facilitate restructuring instead of formal bankruptcy proceedings and so it may have reduced the number of bankruptcies. The change in the law was effected in February 1993 (Virolainen 2004).

FIGURE 1 Monthly industry-specific default rates in the Finnish economy



3.2 Macroeconomic indicators considered

Numerous macroeconomic indicators are usually considered as determinants of corporate default rates. The determinants most frequently mentioned in studies are GDP and interest rates. In the case of GDP, the first difference of real GDP or the deviation from the real GDP trend computed using the Hodrick-Prescott filter[3] can be used.

$$\texttt{GDPdif} = \frac{\texttt{GDP} - \texttt{GDP}_{\texttt{HP}}}{\texttt{GDP}}, \tag{1}$$

where $\texttt{GDP}$ is real GDP and $\texttt{GDP}_{\texttt{HP}}$ is calculated using the Hodrick-Prescott filter. The GDP data are available on a quarterly basis. Monthly GDP data were obtained by disaggregation.[4] We considered the 1M, 3M and 12M HELIBOR, and from 1999 we took the EURIBOR into account. Nominal and real interest rates were investigated. Real interest rates were calculated as

$$r = \frac{1+R}{1+\rho} - 1, \tag{2}$$

[3] The Hodrick-Prescott filter is a smoothing method that is widely used among macroeconomists to obtain a smooth estimate of the long-term trend component of series. The method was first used in a working paper (circulated in the early 1980s and published in 1997) by Hodrick and Prescott to analyze the postwar U.S. business cycle. Technically, the Hodrick-Prescott (HP) filter is a two-sided linear filter that computes the smoothed series s of y by minimizing the variance of y around s, subject to a penalty that constrains the second difference of s. That is, the HP filter chooses s to minimize

$$\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}((s_{t+1} - s_t) - (s_t - s_{t-1}))^2.$$

The penalty parameter λ controls the smoothness of the series σ.

[4] GDP was disaggregated from the quarterly data using EKTA (Bank of Finland software).

where r is the real interest rate, R is the nominal interest rate and ρ is inflation during the appropriate time period. Inflation was expressed by CPI and PPI indexes.[5] The nominal dollar-euro exchange rate was used.[6] The Finnish markka was used before the introduction of the euro in Finland.

Loans to corporations and entrepreneurs were available for the time period 1989–1992 as annual time series and for the time period 1993–2004 as quarterly time series. We constructed the debt indicator as the ratio of outstanding loans to corporations and entrepreneurs to value added in the specific industry (GDP was used in the case of the aggregate economy). Formally,

$$\mathtt{DEBT} = \frac{\mathtt{LOANS}}{\mathtt{GDP_i}}, \tag{3}$$

where $\mathtt{LOANS}$ represents outstanding loans to corporations and entrepreneurs and $\mathtt{GDP_i}$ represents value added in the sector $\mathtt{i}$. It was available from January 1990 after disaggregation to monthly data.[7]

In our analysis, the monthly growth rates of monetary aggregates M1 and M2 were considered. Furthermore, we took into account monthly data on the unemployment rate, the consumer confidence index and the state budget as a percentage of GDP.

4. Macroeconomic credit risk models

The aim of this paper is to find a suite of macroeconomic models for default rate prediction and to investigate the relationship between macroeconomic indicators and the default rate using these models. In general, we want to estimate the function

$$d_{t_1} = f(I_{t_2}), \tag{4}$$

where d_{t_1} is the default rate at time t_1 and $f(I_{t_2})$ is some function of macroeconomic indicators at time $t_2 \leq t_1$. The relationship between the default rate and the macroeconomic indicators can be modeled by this function.

These types of models are usually related to individual risk models, which can be expressed by the following general equation.

$$p_{t_1} = f(X_{t_2}), \tag{5}$$

where p_{t_1} is the individual default probability at time t_1 and X_{t_2} are some indicators of client quality related to financial statements in the case of traditional models, firm value and leverage in the case of structural models, or bond prices in the case of reduced models. Macroeconomic indicators are part of these inputs for all these types of models. Originally, macroeconomic factors were not considered, but in recent years a lot of papers have investigated the influence of the macroeconomic environment on the credit risk model. This issue is currently gaining in importance.

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[5] We used the actual annual inflation rate. Ideally, the expected inflation rate should be used, but data on inflation expectations were not available.

[6] The real effective exchange rate might have been better, but only the nominal exchange rate was available.

[7] New loans to businesses is another possible approach to the debt indicator construction, but these data were not available for the appropriate time period. However, total outstanding loans may be important for explaining default rates in the economy.

Some empirical macroeconomic models may be found in the literature. These models are based on the same idea as the traditional models. They try to find the empirically observed relationship between the default rate and some macroeconomic indicators. This relationship is usually modeled very simply by linear, probit or logit models. Static or dynamic approaches are applied for the modeling. Vector autoregressive models (VAR) are often used in the case of dynamic models. These models are able to model relationships between time series even in the case of time series nonstationarity. Vector autoregressive models can be applied to nonstationary time series if cointegration exists. The vector error correction model (VECs) is able to distinguish between long-run and short-run dependence. The VEC model is just a reformulation of the VAR model.

Another different approach is derived from the Merton model (structural model). This model is employed in the Basel II framework for risk weight calibration and is based on modeling of asset returns. A default event is defined as a fall in a borrowers asset returns below some threshold. This model was originally used for the estimation of individual risks, but recently this idea has been extended to default rate estimation.

4.1 Dynamic model

Empirical models try to estimate the empirical relationship between default rates and some macroeconomic indicators. The exact microeconomic substantiation is not important in this case. Such models explain default rates using some simple function, which is estimated on observed data. Linear, probit or logit models are usually used. A simple static approach can be used, but dynamic models are better for investigating mutual relationships. In case of traditional dynamic models, investigation of the stationarity of the time series used is essential. Vector autoregressive models (VARs) can be used. Their reformulation into the form of the vector error correction model is able to separate long-term and short-term dependence. VAR models are a generalized form of the simple autoregressive process for *n* variables. These models are able to investigate the relationships between variables which are assumed to be random and simultaneously independent. The maximum time lag is known and assumed be the same for all the variables considered.

A linear *l* dimensional autoregressive process of order *p* VAR(p) is defined by equation (6).

$$Y_t = c + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \varepsilon_t, \tag{6}$$

where c is an l dimensional vector of constants, $A_1, \ldots, A_p$ are an $l \times l$ dimensional matrix of parameters, and (ε_t) is an l-dimensional Gaussian white noise process.

The VEC(p) model can be obtained by reformulating VAR(p).

$$\triangle Y_t = c + \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Theta_i \triangle Y_{t-i} + \varepsilon_t, \tag{7}$$

where

$$\Pi = \sum_{i=1}^{p} A_i - I,$$

$$\Theta_i = -\sum_{j=i+1}^{p} A_j.$$

Long-term relationships are expressed by non-differentiated processes and short-term relationships by differentiated (stationary) processes.

We have started to investigate the relationship between credit default rates and macroeconomic indicators using linear vector autoregressive models. However, our target is not to detect the exact relationships between the variables, but only to identify the directions of influence. The exact relationship has been estimated by a more advanced approach derived from Merton's idea.

First, the stationarity of the time series was examined using Dickey-Fuller tests (see Table 23 in the appendix). Different stationarity orders are observed for the default rate time series for agriculture and the other economic sectors. The time series of default rates in Agriculture is integrated order zero while the default rate in whole economy is integrated order one, and the default rate in manufacturing, trade, construction, and transport is also I(1).[8] The default rates in agriculture and transport seem very similar. However, they have different orders of stationarity (see Figure 1). The time series of default rates in construction, manufacturing, and trade have a very similar character. Non-stationary time series can only be used in VAR models when they are cointegrated.

GDP and interest rates are often mentioned in studies, so we investigated the relationships between corporate default rates, GDP and interest rates in the case of the dynamic model. The relationships can be modeled using the VAR or VEC model. We used the first difference of real GDP and the deviation from the real GDP trend. 1M, 3M and 12M nominal and real interest rates were investigated. The order of stationarity is reported in the appendix (see Table 24).

Long-term and short-term relationships can be separated using the VEC model. Long-term relationships are represented by matrix Π in (7). Non-stationary time series can be used for this type of model when they are cointegrated. We investigated the cointegration of default rates, interest rates, and GDP using Johansen's test (Bierens 2004). Our tests showed cointegration of the default rate, interest rates, and GDP. The time series of GDP and interest rates are also cointegrated. This is significant for agriculture, where the default rate time series is stationary. These results show that the original time series of default rates, GDP and interest rates can be used in the VAR or VEC model.

TABLE 1 VAR(2) models

Model	R^2		
$\mathtt{df}$,$\mathtt{dGDP}$,$\mathtt{R1M}_{\mathtt{CPI}}$	0.800894	0.985970	0.959093
$\mathtt{df}_{\mathtt{AGR}}$,$\mathtt{GDPdif}$,$\mathtt{R12M}_{\mathtt{CPI}}$	0.060148	0.998125	0.974966
$\mathtt{df}_{\mathtt{CON}}$,$\mathtt{dGDP}$,$\mathtt{R12M}_{\mathtt{CPI}}$	0.654523	0.985846	0.974692
$\mathtt{df}_{\mathtt{MAN}}$,$\mathtt{dGDP}$,$\mathtt{R1M}_{\mathtt{CPI}}$	0.321865	0.903338	0.135174
$\mathtt{df}_{\mathtt{TRD}}$,$\mathtt{dGDP}$,$\mathtt{R3M}_{\mathtt{PPI}}$	0.280505	0.902341	0.094386
$\mathtt{df}_{\mathtt{TRN}}$,$\mathtt{dGDP}$,$\mathtt{R1M}_{\mathtt{PPI}}$	0.442391	0.887201	0.035169

Table 1 shows the results of the VAR(2) model estimation. The poor performance of the VAR(2) model in estimating the default rate in agriculture is caused by dif-

[8] According to the economic theory, default rates should be stationary in the long run. However, in the 1990s we can observe a significant decreasing trend in many countries. The credit portfolio improvement may be due to progress in risk management techniques.

ferent behavior of the agriculture default rate time series. Agriculture is probably more independent of the cycle of the overall economy. GDP should be replaced by an industry-specific value edited to improve the VAR(2) models for industry-specific sectors. The VAR(2) models of the relationships between default rates, GDP and interest rates were selected as the models with the highest coefficient of determination for default rates. Two options for GDP were considered – the deviation of real GDP from the long-term trend, and the first difference of the real GDP time series. Nominal and real interest rates were considered in the case of interest rates. 1-month, 3-month and 12-month interest rates were examined. The consumer price index (CPI) and producer price index (PPI) were used for the real interest rate calculation. Cointegration relationships for selected models are introduced in Table 2.

TABLE 2 Cointegration relationships between default rates, GDP and interest rates

Model	df	GDP	r
$\mathtt{df,dGDP,R1M}_{\mathtt{CPI}}$	1.00000	0.000000274	-0.000113
$\mathtt{df}_{\mathtt{CON}}\mathtt{,dGDP,R12M}_{\mathtt{CPI}}$	1.00000	0.000000854	-0.000155
$\mathtt{df}_{\mathtt{MAN}}\mathtt{,dGDP,R1M}_{\mathtt{CPI}}$	1.00000	0.000000809	-0.000199
$\mathtt{df}_{\mathtt{TRD}}\mathtt{,dGDP,R3M}_{\mathtt{PPI}}$	1.00000	-0.0000000775	-0.000219
$\mathtt{df}_{\mathtt{TRN}}\mathtt{,dGDP,R1M}_{\mathtt{PPI}}$	1.00000	0.000000195	0.0000108

Johansen cointegration tests showed one cointegration relationship for the selected models. Similar results were obtained for the aggregate economy, construction, and manufacturing. In these cases, the default rates are proportional to interest rates and not proportional to GDP. The values of the cointegration vectors are very close. In the case of trade, the value of the cointegration vector demonstrates a proportional relationship with interest rates as well as GDP. However, a very low cointegration coefficient for GDP reveals an insignificant relationship between default rates and GDP for this sector of the economy. The coefficient for interest rates is very similar to those for the aggregate economy, construction, and manufacturing, but its value is a little higher. In the case of transport, the results show a non-proportional relationship between the default rate and GDP and interest rates. The coefficient of the relationship with GDP is very similar to those for the aggregate economy, construction, and manufacturing, but the low value of the interest rate coefficient demonstrates its insignificance. In the case of agriculture, the time series of default rates is stationary.

Due to the lower performance of VAR(2) for specific sectors, monthly time series of value added for AGR, CON, MAN, TRD, and TRN were used. First, we examined the stationarity of the value added time series. The results of Dickey-Fuller tests are presented in the appendix (see Table 25).

All the value added time series examined were I(1), except for agriculture. The time series of value added in agriculture is stationary and it seems there is no cyclical behavior in this sector. In the case of agriculture, the stationarity of the difference between value added and the long-term trend was also examined, but the result was the same as for the first difference of this time series. Replacing GDP by value added did not improve the performance of the VAR(2) models considered, except in the case of agriculture (see Table 3).

TABLE 3 VAR(2) model with value added for agriculture

Model	R^2		
df_{AGR},$dGDP_{AGR}$,$R12M_{CPI}$	0.483921	0.706177	0.202589

Although such models are able to investigate the relationship between macroeconomic indicators, they are not very good for aggregate default rate estimation, owing to nonlinearity. From now on, we focus on Merton-type models.

4.2 One-factor model

One of the variants of the latent factor model is described by the following equations. This model can be used for the aggregate data which we had available for the Finnish economy. Applications of this model to the German economy can be found in (Rösch 2003) and (Hamerle, Liebig, Scheule 2004). This model is employed by the Basel II Accord. The following model appears in many papers, for example (Rösch 2005), (Céspedes, Martín 2002), (Cipollini, Missaglia 2005) and (Lucas, Klaassen 2003).

The basic idea is based on the Merton model. A normal distribution process is assumed for firm logarithmic asset returns. The discrete normalized logarithmic return process satisfies the following equation for every company in the economy.

$$R_{it} = \sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} \quad (8)$$

R denotes the normalized logarithmic asset return for each firm i at time t. F represents the normalized logarithmic return in the economy independent of firm at time t. This return is assumed to have standard normal random distribution. It can be explained as the macroeconomic-specific part of the return. U denotes the firm-specific return. Standard normal random distribution is assumed. All random variables are assumed to be serially independent.

$$F_t \sim N(0,1)$$

$$U_{it} \sim N(0,1)$$

Coefficient ρ expresses the correlation between the normalized asset returns of any two borrowers.

$$E(R_{it}) = 0 \quad (9)$$

$$Var(R_{it}) = E(R_{it}^2) - E(R_{it})^2 = E(\rho F_t^2 + (1-\rho) U_{it}^2 + 2\sqrt{\rho}\sqrt{1-\rho} F_t U_{it}) = 1 \quad (10)$$

According to the assumption adopted, the asset return for each firm i at time t is standard normal random distributed (9)(10). The basic idea of this model is derived from the Merton model. A default event is assumed to occur when the asset return falls below some threshold. Formally,

$$P(Y_{it} = 1) = P(R_{it} < T), \quad (11)$$

where Y denotes a random variable with two potential states.

$$Y_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults at time } t \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

T can be assumed to be either constant or variable over time. In the latter case, the change in this threshold is considered with regard to change in the macroeconomic environment over time. Various macroeconomic indicators can be considered. Formally,

$$T = \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt}, \tag{13}$$

where x_j represents the j-th macroeconomic indicator and β are constant coefficients. A simple linear relation for the value of the threshold is considered. Change in the macroeconomic conditions affects the value of the default threshold over time. This value is probably higher in good times and lower in bad times. Generally, recessions decrease the value of the threshold for default events. The default probability of firm i at time t is given by equation (14) in the case of a constant default threshold over time.

$$p_i = P(Y_{it} = 1) = P(R_{it} < T) = P(\sqrt{\rho}F_t + \sqrt{1-\rho}U_{it} < \beta_0) = \phi(\beta_0), \tag{14}$$

where ϕ is a function of the cumulative standard normal distribution. In general, other distribution functions can be used; for example, the logistic distribution can be assumed (Hamerle, Liebig, Scheule 2004). The conditional default probability on realization f_t of a random factor at time t can be described by the following formula:

$$p_i(f_t) = P(U_{it} < \frac{\beta_0 - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}}) = \phi\left(\frac{\beta_0 - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}}\right) \tag{15}$$

The default probability of firm i at time t is given by equation (16) in the case where a change in the threshold is considered according to equation (13).

$$p_{it} = P(Y_{it} = 1) = P(R_{it} < T) = P(\sqrt{\rho}F_t + \sqrt{1-\rho}U_{it} < \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt}) = \tag{16}$$

$$= \phi(\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt}).$$

The conditional default probability on realization f_t of the random factor and the macroeconomic indicators x_t at time t can be obtained in this case from formula (17).

$$p_i(f_t) = P(U_{it} < \frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}}) = \phi\left(\frac{\beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} - \sqrt{\rho}\, f_t}{\sqrt{1-\rho}}\right) \tag{17}$$

The same result is obtained under the assumption that the macroeconomic indicators are considered as part of the factor of asset return independent of firm i at time t. This concept is used, for example, in (Hamerle, Liebig, Scheule 2004). Formally,

$$R_{it} = \alpha F_t + \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} + \omega U_{it}. \tag{18}$$

If the number of borrowers in the portfolio is assumed to be very high, all counterparties have the same individual probability p_i, and all default events are independent, then according to the law of large numbers the default rate on the portfolio can be estimated as the individual default probability.

$$P(p(f_t) = p_i(f_t)|F_t = f_t) = 1 \tag{19}$$

The unconditional default probability can be obtained by

$$p = P(Y_t = 1) = \int_{-\infty}^{\infty} P(Y_t = 1|F_t = f_t)\psi(f_t)d f_t = \int_{-\infty}^{\infty} p(f_t)\psi(f_t)d f_t, \tag{20}$$

where ψ is a function of the standard normal distribution.

The random factor is assumed independent between borrowers. The number of defaults $D_t(f_t)$ at time t has binomial distribution with a conditional default probability $p(f_t)$ and a given number of companies N_t.

$$D(f_t) \sim \mathrm{Bi}(N_t, p(f_t)) \tag{21}$$

The conditional probability of having exactly d_t defaults at time t can be expressed as

$$P(D_t = d_t|F_t = f_t) = \binom{n_t}{d_t} p(f_t)^{d_t}(1 - p(f_t))^{n_t - d_t}. \tag{22}$$

The unconditional probability of having exactly d_t defaults at time t can be expressed as

$$P(D_t = d_t) = \int_{-\infty}^{+\infty} \binom{n_t}{d_t} p(f_t)^{d_t}(1 - p(f_t))^{n_t - d_t}\psi(f_t)d f_t. \tag{23}$$

4.3 One-factor model estimation

The parameters of model (15) or (17) can be estimated by a log-likelihood function. The number of defaults D_t is a conditional binomial distributed random variable with number of borrowers N_t and conditional probability $p(f_t)$ according to equation (21). The default numbers d_t are observed. The realizations d_t and n_t of random variables D_t and N_t are known, $d_t = \sum_{i=1}^{n_t} d_{it}$.

The unconditional number of defaults can be computed by integrating over the random effects (20). The log-likelihood function depends only on parameters β and ρ. Formally for model (15):

$$l(\beta, \rho) = \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{\infty} \binom{n_t}{d_t} \phi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)^{d_t} \right. .$$

$$\cdot \left[1-\phi\left(\frac{\beta_0-\sqrt{\rho}\,f_t}{\sqrt{1-\rho}}\right)\right]^{n_t-d_t}\psi(f_t)d\,f_t\Bigg\} \tag{24}$$

The log-likelihood function for model (17) is similarly given by equation (25).

$$l(\beta_0,\cdots,\beta_N,\rho)=\sum_{t=1}^{T}\ln\left\{\int_{-\infty}^{\infty}\binom{n_t}{d_t}\phi\left(\frac{\beta_0+\sum_{j=1}^{N}\beta_j x_{jt}-\sqrt{\rho}\,f_t}{\sqrt{1-\rho}}\right)^{d_t}\cdot\right.$$

$$\left.\cdot\left[1-\phi\left(\frac{\beta_0+\sum_{j=1}^{N}\beta_j x_{jt}-\sqrt{\rho}\,f_t}{\sqrt{1-\rho}}\right)\right]^{n_t-d_t}\psi(f_t)d\,f_t\right\} \tag{25}$$

4.4 Multi-factor model

These types of models are generalized versions of the one-factor model. Multi-factor models assume M correlated factors in the economy. The multi-factor model framework can be interpreted as a world of M economies or countries where factors are common for all firms in the relevant economy or country. These M economies are related, because there is correlation between factors. A two-factor model is discussed for example in (Céspedes, Martín 2002). A continuous version of a three-factor model can be found in (Hui, Lo, Huang 2003).

In the case of model 8, one can generalize to multi-factor models using the following equations.

$$R_{it}^{1}=\sqrt{\rho_1}F_t^{1}+\sqrt{1-\rho_1}U_{it}^{1}$$

$$\cdots$$

$$R_{it}^{M}=\sqrt{\rho_M}F_t^{M}+\sqrt{1-\rho_M}U_{it}^{M} \tag{26}$$

$$f_i=\rho_{ij}f_j+\sqrt{1-\rho_{ij}^2}\eta_{ij}\quad\forall i,j\in\{1,\cdots,M\},\ i\neq j$$

$$\rho_{ij}=\mathrm{corr}(f_i,f_j)\quad\forall i,j\in\{1,\cdots,M\},\ i\neq j \tag{27}$$

where $f_1\cdots f_M,\ \eta_{ij}\ \forall i,j\in\{1,\cdots,M\},\ i\neq j$ are N(0,1) i.i.d. The conditional default probability can be derived for each country similarly as in the case of the one-factor model. The conditional default probability satisfies the following equations.

$$p_i^1(f_t^1)=\phi\left(\frac{T_1-\sqrt{\rho_1}\,f_t^1}{\sqrt{1-\rho_1}}\right)$$

$$\cdots$$

$$p_i^M(f_t^M)=\phi\left(\frac{T_M-\sqrt{\rho_M}\,f_t^M}{\sqrt{1-\rho_M}}\right) \tag{28}$$

where $T_1\cdots T_N$, is the value of the threshold, which can be modeled either as constant over time or as a random variable, as in the case of the one-factor model.

$\rho_1, \cdots, \rho_M$ are constants representing the correlation between firm assets in each economy or country. Due to the independence of all default events, the portfolio default probability can be modeled by the weighted sum of default in each segment. The law of large numbers can be applied. The default rate in each segment can be estimated as the individual probability of the firm in the specific segment. The default rate in the portfolio is estimated by the default rates in the segments weighted by the fractions of the segments in the portfolio. Formally,

$$P(p(f_t) = w_t^1 p_i^1(f_t^1) + \cdots + w_t^M p_i^M(f_t^M)) = 1, \tag{29}$$

where $w_t^1, \cdots, w_t^N$ represent the fraction of the specific segment in the portfolio at time t. Formally,

$$w_t^i = N_t^i / N_t, \tag{30}$$

where N_t^i denotes the number of firms in the i-th specific economy at time t and N_t denotes the number of firms in the portfolio at time t.

The number of defaults $D^i(f_t^i)$ is binomial distributed within the specific segment of economy.

$$\begin{gathered} D^1(f_t^1) \sim Bi(N_t^1, p^1(f_t^1)) \\ \cdots \\ D^M(f_t^M) \sim Bi(N_t^M, p^M(f_t^M)) \end{gathered} \tag{31}$$

The conditional probability of having exactly d_t defaults at time t in all economy can be expressed as the product of the conditional probabilities for the industry-specific sectors, due to the independence of random events within segments as well as between segments.

$$\begin{gathered} P(D_t = d_t | F_t = f_t) = \sum_{s_1=0}^{d_t} \binom{n_t^1}{s_1} p^1(f_t)^{s_1} (1 - p^1(f_t))^{n_t^1 - s_1} \cdot \\ \cdot \sum_{s_2=0}^{d_t - s_1} \binom{n_t^2}{s_2} p^2(f_t)^{s_2} (1 - p^2(f_t))^{n_t^2 - s_2} \cdots \\ \cdots \sum_{s_M=0}^{d_t - s_{M-1} - s_{M-2} - \cdots - s_1} \binom{n_t^M}{s_M} p^1(f_t)^{s_M} (1 - p^M(f_t))^{n_t^M - s_M} \end{gathered} \tag{32}$$

Equation (32) is valid for $d_t \leq n_i$, for all $i \in \{1, \cdots, M\}$. For other cases, equation (32) should be adjusted. This assumption is very realistic in our case. We want to model default for industry-specific sectors of the economy. The number of defaults in the whole economy is very small compared to the number of firms in the five industry-specific sectors we consider (AGR, CON, MAN, TRD, TRN).

The unconditional probability of having exactly d_t defaults at time t can be expressed as

$$P(D_t = d_t) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} P(D_t = d_t | F_t = f_t) \psi(f_t^1, \cdots, f_t^M) d f_t^1 \cdots d f_t^M. \tag{33}$$

4.4.1 Multi-factor model estimation

The parameters of model (28) can be estimated similarly as for the one-factor model. However, the likelihood function is more complicated in the case of the multi-factor model.

$$
\begin{aligned}
l(\beta^1,\cdots,\beta^M,\rho^1,\cdots,\rho^M) = \sum_{t=1}^{T} \ln\Big\{ \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \sum_{s_1=0}^{d_t} \binom{n_t^1}{s_1} p^1(f_t)^{s_1}(1-p^1(f_t))^{n_t^1-s_1} \\
\cdot \sum_{s_2=0}^{d_t-s_1} \binom{n_t^2}{s_2} p^2(f_t)^{s_2}(1-p^2(f_t))^{n_t^2-s_2} \\
\cdots \sum_{s_M=0}^{d_t-s_{M-1}-s_{M-2}-\cdots-s_1} \binom{n_t^M}{s_M} p^1(f_t)^{s_M}(1-p^M(f_t))^{n_t^M-s_M}\psi(f_t^1,\cdots,f_t^M)\,df_t^1\cdots df_t^M\Big\}
\end{aligned}
\tag{34}
$$

The multi-factor models assume that data on default numbers d_t^i and numbers of firms n_t^i $\forall i \in \{1,\cdots,M\}$ are observed in each specific sector of the economy separately.

5. Results of a latent-factor model: the Finnish economy

5.1 Data

Data on bankruptcies are used to estimate a one-factor model. Specifically, a monthly time series of firms' bankruptcies and a yearly time series of firms' numbers were used. Data on the numbers of firms were disaggregated from the annual data.[9] GDP, interest rates, debt ratios and exchange rates were used as the macroeconomic indicators in the models (17). Lagged macroeconomic variables were also tested, but only the lagged exchange rate was significant in the case of the latent one-factor model. The other macroeconomic indicators were significant only as non-lagged variables. All calculations were based on monthly data.

5.2 Model

We started by estimating the one-factor model for the aggregate economy. Constant correlation between the normalized asset returns of firms is assumed. This model can provide better results for a relatively homogeneous portfolio. Due to this fact, industry-specific sectors were considered. We estimated the one-factor model separately for each industry-specific sector (AGR, MAN, CON, TRD, TRN). Unfortunately, this model is not able to give adequate results for the relationships between industry-specific sectors. A multi-factor model could be better as regards providing some results on the interactions between industry-specific sectors. This kind of model follows the relationships between sectors by means of correlation parameters for industry-specific factors. However, estimating multi-factor models is numerically fairly complicated. We had access to data for five industry-specific sectors, which means a five-factor model would have to be used. Only the estimation of the one-sector model separately

[9] The numbers of firms were disaggregated from the annual data using EKTA (Bank of Finland software).

for each industry-specific sector has been performed. Model (17) was estimated for the aggregate economy and also for each industry-specific sector. This model follows the relationships between the default rate and macroeconomic indicators and can be use for stress testing as well.

5.3 Aggregate economy

Models (15) and (17) were estimated for the Finnish economy for the data used. Both of the models were also re-estimated for the data starting in January 1993, due to the change in the bankruptcy law in 1993. The results obtained were compared.

Table 4 shows the estimation of model (15) for the data starting in January 1988. The constant parameter β_0 was estimated as -2.9528. It corresponds to a default probability of about 0.16%. The estimated correlation between the normalized asset returns of borrowers is about 1.7%. It corresponds to a 12-month correlation between the normalized asset returns of borrowers of about 5.7%. Both coefficients were highly significant. The 12-month default probability corresponds to an estimated monthly default probability of about 1.89% under the constant default assumption.

TABLE 4 Estimation of (15) for data starting in January 1988 (aggregate economy)

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-2.9528	0.009731	$< .0001$
ρ	0.01659	0.001701	$< .0001$

Table 5 shows the estimation of model (15) for the data starting in January 1993. The constant parameter β_0 was estimated as -2.9699. It corresponds to a default probability of about 0.15%. The estimated correlation between the normalized asset returns of borrowers is about 1.5%. It corresponds to a 12-month correlation between the normalized asset returns of borrowers of about 5.7%. Both coefficients were highly significant. The 12-month default probability corresponds to an estimated monthly default probability of about 1.79% under the constant default assumption. One can see very similar results in both cases. We can conclude that the model is fairly robust to the change in the bankruptcy law in 1993.

TABLE 5 Estimation of (15) for data starting in January 1993 (aggregate economy)

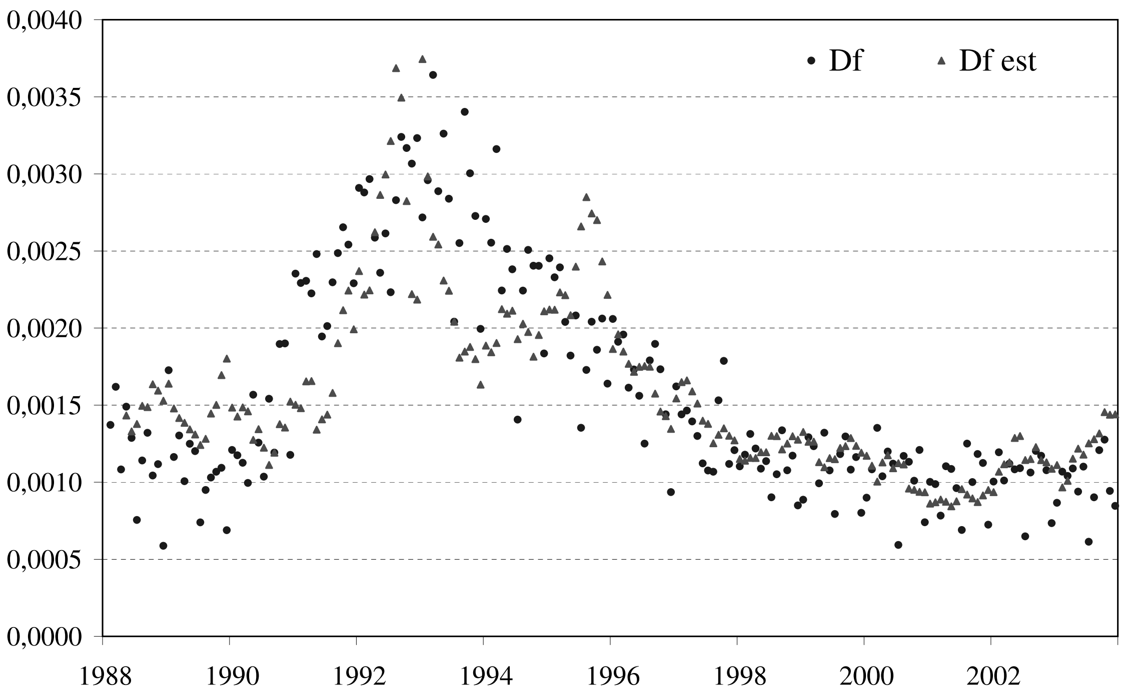
Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-2.9699	0.01118	$< .0001$
ρ	0.01518	0.001877	$< .0001$

Table 6 shows the estimation of model (17) for the data starting in January 1988. GDP (β_1), the interest rate (β_2) and the exchange rate (β_3) were used as the macroeconomic indicators in this calculation. These estimations confirmed the theory of a negative relationship between GDP and the default probability and a positive relationship between interest rates and the default probability. A dummy variable (β_4) was used to allow for the bankruptcy law change in 1993. The values of this variable are zero until the end of 1992 and one from the beginning of 1993. The deviation of real GDP computed according to equation (1) was considered. Interest rates (R) were represented by the real 12-month interest rate computed according to equation (2). The exchange rate (ER) is represented by the nominal dollar-euro exchange rate. The Finnish markka was used before the introduction of the euro in Finland. According to this model, there is a positive relationship between the default rate and the nominal dollar-euro exchange rate. The four-month-lagged variable of the exchange rate was used. The estimated unobservable factor coefficient is about 0.7%. All the coefficients were significant at the 5% confidence level. Figure 2 shows the performance of the estimated model (17) for the data starting in January 1988.

TABLE 6 Estimation of (17) for data starting in January 1988 (aggregate economy)

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.5085	0.06804	$< .0001$
β_1 (GDP)	-0.04348	0.005699	$< .0001$
β_2 (R)	0.05427	0.004450	$< .0001$
β_3 (ER_{t-4})	0.1171	0.05064	0.0219
β_4 (DUMMY)	0.2426	0.02590	$< .0001$
ρ	0.006827	0.000735	$< .0001$

FIGURE 2 Performance of the one-factor model for the Finnish economy



We tried to re-estimate the model for the data starting in January 1993. Table 7 shows that the results as regards the relationship between the default rate, GDP, and interest rates were fairly similar, but the relationship between the default rate and the exchange rate was different. Because of the insignificance of the exchange rate coefficient (β_3) in the case of the model estimation for the data starting in January 1993, we can conclude a weak or unstable relationship between the exchange rate and the default rate over time. Furthermore, we can conclude that the relationship between the default rate, GDP, and interest rates is quite stable over time.

TABLE 7 Estimation of (17) for data starting in January 1993 (aggregate economy)

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.0971	0.05112	$< .0001$
β_1 (GDP)	-0.05478	0.007379	$< .0001$
β_2 (R)	0.06537	0.004971	$< .0001$
β_3 (ER_{t-4})	-0.06831	0.05256	0.1960
ρ	0.004806	0.000632	$< .0001$

Furthermore, we tried to add some indicators of debt to the model, due to the Merton concept of default event. We constructed the debt indicator as the ratio of outstanding loans to corporations and entrepreneurs to GDP according to equation (3). It was available from January 1990 after disaggregation to monthly data. We had to restrict the start of all our time series to January 1990 due to the limited debt indicator time series. The following Table 8 demonstrates the estimated model (17) with the debt indicator (DEBT).

TABLE 8 Estimation of (17) with debt indicator for data starting in January 1990 (aggregate economy)

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.3969	0.04896	$< .0001$
β_1 (GDP)	-0.04114	0.004281	$< .0001$
β_2 (R)	0.01587	0.004527	0.0006
β_3 (ER_{t-4})	0.06670	0.03612	0.0666
β_4 (DEBT)	0.1767	0.01629	$< .0001$
β_5 (DUMMY)	0.1187	0.02154	$< .0001$
ρ	0.003097	0.000374	$< .0001$

The debt indicator is highly significant in the estimated model. This model can better explain the default rate than the model without the debt indicator. The estimation proved a positive relationship between the default rate and the debt indicator. The exchange rate is not significant at the 5% confidence level. We can re-estimate this model with the debt indicator and without the exchange rate. Table 9 shows the results of the re-estimated model. All the coefficients are highly significant.

Figure 3 shows the performance of the estimated model (17) for the data starting in January 1990 with the debt indicator and without the exchange rate (see Table 9).

The one-factor model assumes constant correlation of the normalized asset returns of borrowers. This assumption can be satisfied in the case of a homogeneous portfolio. For this reason, the following analysis was focused on the industry-specific sectors.

TABLE 9 Estimation of (17) with debt indicator for data starting in January 1990 (aggregate economy)

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.3222	0.02794	$< .0001$
β_1 (GDP)	-0.04027	0.004301	$< .0001$
β_2 (R)	0.01802	0.004424	$< .0001$
β_4 (DEBT)	0.1795	0.01639	$< .0001$
β_5 (DUMMY)	0.1092	0.02113	$< .0001$
ρ	0.003170	0.000382	$< .0001$

5.4 Agriculture

The results of the one-factor model (17) for agriculture (see Table 10) show a significant influence of the latent factor in the model. Coefficient ρ is significant at the 1% confidence level. Contrary to the empirical model (chapter 4), the results of the one-factor model show a negative relationship between the default rate and GDP at the 5% confidence level. Exchange rates and interest rates are probably insignificant for default events in the agriculture sector. Due to the insignificant coefficient β_4, the bankruptcy law change probably did not affect the default level in the agriculture sector.

TABLE 10 Estimation of (17) for agriculture

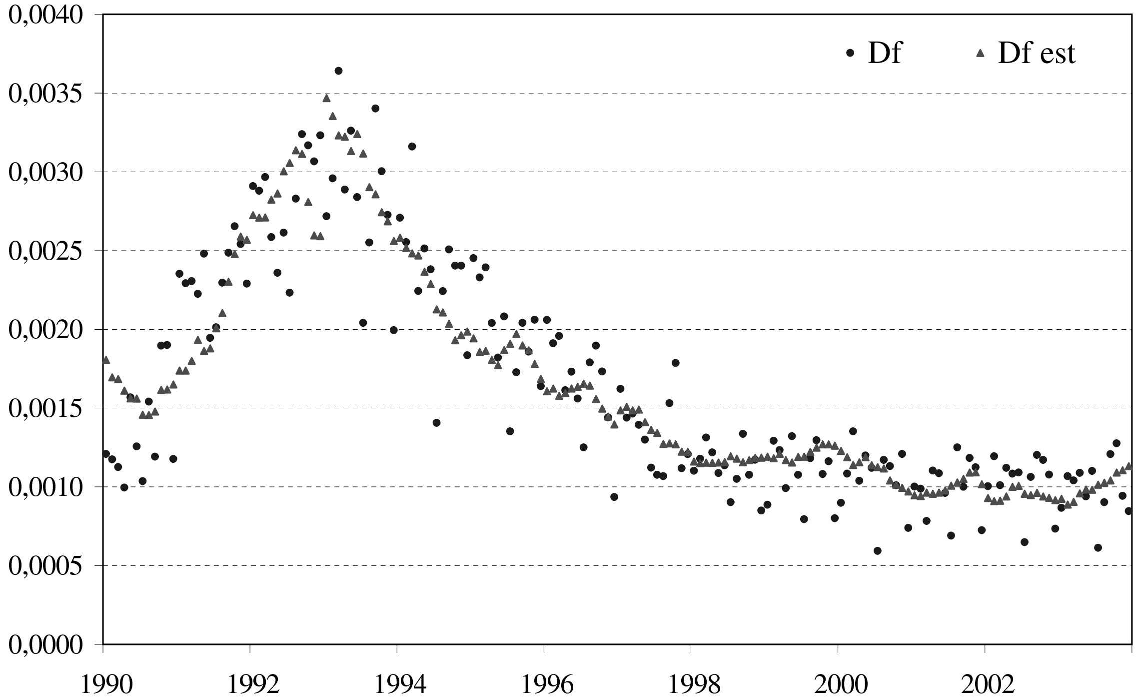
Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.4311	0.1300	$< .0001$
β_1 (GDP)	-0.02653	0.01097	0.0165
β_2 (R)	-0.00319	0.008534	0.7089
β_3 (ER_{t-4})	0.1354	0.09641	0.1617
β_4 (DUMMY)	0.04148	0.04937	0.4019
ρ	0.008009	0.002649	0.0029

The following Table 11 shows the results of the one-factor model (17) where debt ratio indicators were considered. Due to this fact, only the time series starting in January 1990 was taken into account. All the macroeconomic indicators are insignificant, in contrast to the results for the model estimated for the data starting in January 1998. The default rate in agriculture can be explained only by unobservable factors in this case, because coefficient ρ was significant at the 1% confidence level.

5.5 Manufacturing

The results of the one-factor model (17) demonstrate similar behavior for the manufacturing sector as for the aggregate economy (see Table 12). However our results show the insignificance of the exchange rate for default rate prediction. The model proved the dependence of the default rate on GDP and interest rates. Both coefficients (β_1, β_2)

FIGURE 3 Performance of the one-factor model with debt indicator for the Finnish economy



were highly significant. The change in the bankruptcy law in 1993 was important for the default rate level in this sector, according to the results achieved (coefficient β_4). The unobserved factor is still highly significant.

Table 13 shows the results of model (17) for manufacturing where debt is taken into account. The coefficient of the dummy variable (β_4) is insignificant in this model. The change in the bankruptcy law is not important in the model when the debt indicator is considered. All the other coefficients are significant at the 5% confidence level.

5.6 Construction

The results of the one-factor model for construction are similar to those for the manufacturing sector (see Table 14). Except for the exchange rate, all the variables included in the model are significant. The exchange rate probably does not play an important role in the default events of firms.

Table 15 summarizes the estimation of model (17) for the construction sector with the inclusion of the debt indicator. All the coefficients are significant at the 5% confidence level. The results proved a positive correlation between default events and indebtedness of corporations and entrepreneurs. GDP, interest rates, and the change in the bankruptcy law were still important for explaining default rates.

5.7 Trade

Table 16 shows the results of the one-factor model (17) for the trade sector. The level of the default rate depends on GDP, interest rates, and the exchange rate in the economy. The coefficient of the exchange rate (β_3) is significant at the 5% confidence

TABLE 11 Estimation of (17) for agriculture (with debt indicator) and data starting in January 1990

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.5185	0.1754	$< .0001$
β_1 (GDP)	-0.01081	0.01292	0.4036
β_2 (R)	-0.00428	0.01025	0.6769
β_3 (DEBT)	0.2080	0.1456	0.1548
β_4 (DUMMY)	0.07308	0.06341	0.2507
ρ	0.007383	0.002756	0.0081

TABLE 12 Estimation of (17) for manufacturing

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.3654	0.08968	$< .0001$
β_1 (GDP)	-0.04865	0.007380	$< .0001$
β_2 (R)	0.06016	0.005745	$< .0001$
β_3 (ER_{t-4})	0.09880	0.06638	0.1383
β_4 (DUMMY)	0.1747	0.03364	$< .0001$
ρ	0.01012	0.001237	$< .0001$

level. All the other coefficients are highly significant. The exchange rate plays an important role in the trade sector due to international business. This model proved this intuitive exception. The change in the bankruptcy law was important for the default rate level in trade, according to this model (coefficient β_4). The unobserved factor is still significant.

Furthermore, we estimated the model where the exchange rate was replaced by the debt indicator (see Table 17). All the coefficients are significant at the 5% confidence level.

5.8 Transport

The following Table 18 demonstrates similar results for transport as we obtained for manufacturing and construction. The default rate depends negatively on GDP and positively on interest rates. Exchange rates are not important for the default rate in transport. All the coefficients except that for the exchange rate are highly significant.

Table 19 shows the estimated model (17) for transport where the debt indicator was considered. In this case, only the debt indicator and the change in the bankruptcy law are important macro indicators explaining the default rate. The unobservable factor is still highly significant.

TABLE 13 Estimation of (17) for manufacturing (with debt indicator) and data starting in January 1990

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.1738	0.03683	$< .0001$
β_1 (GDP)	-0.04184	0.005695	$< .0001$
β_2 (R)	0.01334	0.005632	0.0190
β_3 (DEBT)	0.05686	0.005316	0.0001
β_4 (DUMMY)	0.04120	0.02726	0.1326
ρ	0.004158	0.000684	$< .0001$

TABLE 14 Estimation of (17) for construction

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.3014	0.07853	$< .0001$
β_1 (GDP)	-0.04505	0.006531	$< .0001$
β_2 (R)	0.04938	0.005086	$< .0001$
β_3 (ER_{t-4})	0.05533	0.05832	0.3440
β_4 (DUMMY)	0.1766	0.02986	$< .0001$
ρ	0.007381	0.000986	$< .0001$

5.9 Comparison of results for industry-specific sectors

Table 20 compares the estimation of model (17) for industry-specific sectors. The marks * and ** denote the significance of the estimation (1% confidence level and 5% confidence level, respectively). Only coefficients significant at the 5% confidence level are given in the table.

The results obtained prove a negative relationship between default rates and GDP for all the sectors of the economy investigated. The estimated coefficients for GDP were quite similar for manufacturing, construction, and trade, but the default rate for manufacturing probably has the strongest relationship to GDP. Similar coefficients were obtained for construction and trade. Both of them were about -0.045. The weakest relationships between default rates and GDP were estimated for the sectors of transport and agriculture. However, these relationships were further tested against empirical models, where a relationship was not proved for agriculture. All of the estimated coefficients for GDP were significant at the 5% confidence level. Except for agriculture, they were significant even at the 1% confidence level.

Interest rates (R) play an important role in default events in the all sectors examined except agriculture. Agriculture is probably not sensitive to changes in the interest rate. The coefficients of interest rates were significant at the 1% confidence level for all the other sectors. Positive relationships between default rates and interest rates were proved. The sector most dependent on the interest rate is probably trade, followed by manufacturing. Conversely, the weakest relationship was obtained for transport. However, the estimated coefficients for interest rates were fairly similar in all sectors except transport.

TABLE 15 Estimation of (17) for construction (with debt indicator) and data starting in January 1990

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.2043	0.03452	$< .0001$
β_1 (GDP)	-0.02573	0.005705	$< .0001$
β_2 (R)	0.01118	0.005346	$< .0381$
β_3 (DEBT)	0.4956	0.05446	$< .0001$
β_4 (DUMMY)	0.06308	0.02571	0.0152
ρ	0.003339	0.000603	$< .0001$

TABLE 16 Estimation of (17) for trade

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.5832	0.08157	$< .0001$
β_1 (GDP)	-0.04550	0.006812	$< .0001$
β_2 (R)	0.06406	0.005301	$< .0001$
β_3 (ER_{t-4})	0.1480	0.06057	0.0155
β_4 (DUMMY)	0.2909	0.03093	$< .0001$
ρ	0.009184	0.001050	$< .0001$

The exchange rate (ER) was important for default events only in the trade sector. The value of the exchange rate plays an important role in this sector probably due to international trade. The nominal dollar-euro exchange rate was considered. However, we cannot reject the exchange rate as an important indicator of default events in other sectors, due to its high correlation with interest rates. In the case of trade, a positive relationship between default rates and exchange rates was proved. The result means fewer default events with a stronger currency. This result is not fully explained by economic theory. The value of the four-month-lagged exchange rate was the most significant.

The change in the bankruptcy law (DUMMY) probably affected the level of default rates in all sectors except agriculture. The coefficients of the dummy variable were significant at the 5% confidence level in the cases of manufacturing, construction, trade, and transport. It seems that the change in this law did not influence the agriculture sector. The construction and trade sectors were affected very similarly, according to their similar estimated coefficients for the dummy variables.

The unobserved factor was significant in all cases. Coefficients ρ were significant for all industry-specific sectors. The values of these coefficients were fairly similar.

Slightly different results are shown in Table 21, which presents the results of the one-factor models for the aggregate economy and industry-specific sectors. Data starting in January 1990 were used for the model estimation. The debt indicator was considered. This model contains GDP, interest rates, the debt indicator, and a dummy variable as a proxy for the change in the bankruptcy law. The marks * and ** have the same meaning as in the previous case. Only significant coefficients are given in

TABLE 17 Estimation of (17) for trade (with debt indicator) and data starting in January 1990

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.3635	0.03419	$< .0001$
β_1 (GDP)	-0.01695	0.006046	0.0057
β_2 (R)	0.01400	0.005696	0.0150
β_3 (DEBT)	0.2546	0.02351	$< .0001$
β_4 (DUMMY)	0.1282	0.02519	$< .0001$
ρ	0.004104	0.000567	$< .0001$

TABLE 18 Estimation of (17) for transport

Parameter	Estimate	Standard error	$Pr > \|t\|$
β_0	-3.4396	0.07669	$< .0001$
β_1 (GDP)	-0.02305	0.0066671	0.0007
β_2 (R)	0.02356	0.005120	$< .0001$
β_3 (ER_{t-4})	0.04380	0.05758	0.4478
β_4 (DUMMY)	0.1957	0.03017	$< .0001$
ρ	0.004561	0.000962	$< .0001$

Table 21.

The results obtained confirm a negative relationship between GDP and default rates only in the case of manufacturing, construction, and trade. However, in the case of transport the coefficient was significant at the 6.27% confidence level. These results show time instability of this relationship in the cases of transport and, most notably, agriculture, where the estimated coefficient was highly insignificant. The strongest relationship was obtained in manufacturing. Default events are probably most affected by recessions in manufacturing. This result corresponds with the previous results (Table 20).

Similar results were obtained for interest rates (R). A positive relationship between interest rates and default rates was proved in the cases of manufacturing, construction, and trade. The strongest relationship was obtained for trade. This result corresponds with the previous results (Table 20).

The debt indicator (DEBT) considered was the ratio of the gross debt of the industry (outstanding loans to corporations and entrepreneurs) to value added in that industry. The coefficients of the indebtedness indicator were significant in all the sectors considered except agriculture. Our hypothesis that indebtedness is an important determinant of default rates has been proved. Positive relationships between indebtedness and default rates in the economy were shown in all sectors except agriculture. Agriculture seems to be independent of, or only slightly dependent on, the macroeconomic environment.

The coefficients of the change in the bankruptcy law (DUMMY) are significant in the cases of construction, trade, and transport. The coefficient is insignificant in

TABLE 19 Estimation of (17) for transport (with debt indicator) and data starting in January 1990

Parameter	Estimate	Standard error	$Pr > \lvert t \rvert$
β_0	-3.4470	0.06308	$< .0001$
β_1 (GDP)	-0.01348	0.007194	0.0627
β_2 (R)	0.009504	0.006553	0.1489
β_3 (DEBT)	0.04651	0.1840	0.0124
β_4 (DUMMY)	0.1586	0.03218	$< .0001$
ρ	0.003202	0.000822	0.0001

TABLE 20 Comparison of models (17) for industry-specific sectors of the economy

Sector	GDP	R	ER_{t-4}	DUMMY	ρ
Aggregate	-0.04348**	0.05427**	0.1171*	0.2426**	0.006827**
Agriculture	-0.02653*	–	–	–	0.008009**
Manufacturing	-0.04865**	0.06016**	–	0.1747**	0.010120**
Construction	-0.04505**	0.04938**	–	0.2986**	0.007381**
Trade	-0.04550**	0.06406**	0.1480*	0.2909**	0.009184**
Transport	-0.02305**	0.02356**	–	0.1957**	0.000962**

the case of manufacturing when the debt indicator was included in the model estimated on data starting in January 1990.

Very similar ρ coefficients were obtained in all cases. These coefficients represent unobservable factors. A slightly different result was estimated for agriculture, where this coefficient is higher due to the insignificance of the macroeconomic variables in the model.

TABLE 21 Comparison of models (17) for industry-specific sectors of the economy

Sector	GDP	R	DEBT	DUMMY	ρ
Aggregate	-0.04027**	0.01802**	0.1795**	0.1092**	0.003170**
Agriculture	–	–	–	–	0.007383**
Manufacturing	-0.04184**	0.01334*	0.05686**	–	0.004158**
Construction	-0.02573**	0.01118*	0.4956**	0.06308*	0.003339**
Trade	-0.01695**	0.01400**	0.2546**	0.1282**	0.004104**
Transport	–	–	0.04651*	0.1586**	0.003202**

The relationships between the various sectors of the economy are apparent from the results of the one-factor models. The relationships can be described by the correlation matrix for the default rates (df) of the industry-specific sectors (agriculture - AGR, manufacturing - MAN, trade - TRD, construction - CON, transport - TRN). The significance of each coefficient is shown in parenthesis. The correlation matrix demonstrates a high correlation between manufacturing, trade, and construction. The default rate of transport is less correlated with the others. One can also see a very low correlation between agriculture and all the other industry-specific sectors.

TABLE 22 Pearson correlation coefficients for the industry-specific default rate

	df_{AGR}	df_{MAN}	df_{TRD}	df_{CON}	df_{TRN}
df_{AGR}	1.00000	0.14754 (0.0445)	0.17953 (0.0142)	0.20101 (0.0059)	0.29201 (< .0001)
df_{MAN}	0.14754 (0.0445)	1.00000	0.91775 (< .0001)	0.88748 (< .0001)	0.45449 (< .0001)
df_{TRD}	0.17953 (0.0142)	0.91775 (< .0001)	1.00000	0.90995 (< .0001)	0.52673 (< .0001)
df_{CON}	0.20101 (0.0059)	0.88748 (< .0001)	0.90995 (< .0001)	1.00000	0.50152 (< .0001)
df_{TRN}	0.29201 (0.0059)	0.45449 (< .0001)	0.52673 (< .0001)	0.50152 (< .0001)	1.00000

6. Conclusion

We have investigated macroeconomic models for default rate estimation. We used two possible approaches. First, empirical models were researched. Second, latent factor models were examined. All the models used are derived from individual risk models. The empirical models are based on the idea of traditional models. This approach assumes the estimation of empirical functions. Linear, logit or probit functions are usually used. Latent factor models are derived from the Merton idea (Merton 1974). These models were originally employed in individual risk modeling. Unobservable factors are used by the latent models in credit risk modeling. Normal distribution of these unobservable factors is usually assumed. A static version of this model was considered for estimation in this paper. The coefficients can be estimated by means of a likelihood function. Solution of the maximization problem leads to an integral over the random effects.

We employed monthly data on the Finnish economy. Bankruptcy data and time series of company numbers were the key time series used. Numerous macroeconomic indicators were considered. In the end, GDP, interest rates, the exchange rate, and company indebtedness were employed in the default rate modeling. Time series starting in January 1988 and finishing in December 2003 were available for all the data considered except indebtedness. Outstanding loans to corporations and entrepreneurs were available only from January 1990. Due to the shorter time series, the indebtedness part of the analysis was restricted to the period January 1990 – December 2003. Yearly and quarterly time series were disaggregated. The whole aggregate economy as well as industry-specific sectors – agriculture, manufacturing, construction, trade, and transport were investigated.

Firstly, linear vector autoregressive models were researched in the case of empirical dynamic models. Industry-specific default rates were investigated. No relationships with the macroeconomic indicators were proved in the agriculture sector. A negative relationship between default rates and GDP was proved in all the other sectors except trade. A positive relationship between default rates and interest rates was proved in all cases except agriculture and transport.

Furthermore, a one-factor model was used for default rate estimation in the aggregate economy and also in industry-specific sectors. A multi-factor model was also considered, but due to the complicated numerical solution of such models, only the one-factor model was estimated. Unobservable factors of this model were significant in all cases. The one-factor model signaled different behavior of the agriculture sector. This sector is probably independent of, or only weakly dependent on, the macroeconomic environment. A negative relationship between GDP and default rates was solidly proved in the cases of manufacturing, construction, and trade. There is probably a weak negative relationship between default rates and GDP in transport. A very similar conclusion with a positive relationship was reached in the case of interest rates, but any relationship between interest rates and default rates in agriculture was rejected. A significant indicator of the default rate is company indebtedness. Positive relationships were proved in all cases except agriculture. The exchange rate probably affects the default rate only in the case of trade, which is exposed to international business.

This research is linked to a study by Virolainen (2004). We tried to improve the suggested default rate model. The (Virolainen 2004) study is based on the logit empirical model. The estimated one-factor model offers an alternative to the empirical model, which has no microeconomic foundations. We used very similar indicators as in the previous research. However, some slight differences can be observed. The previous study did not find any role of real interest rates. By contrast, real interest rates were employed in our model and a significant strong relationship was proved at least in the cases of manufacturing, construction, and trade. The agriculture sector is less affected by macroeconomic indicators according to our study than according to the previous study. This problem may be related to regression, because time series stationarity was not investigated in the previous study. However, all significant relationships in both studies have the same sign.

Some aspects of the latent factor model could be further elaborated. Different assumptions about the default distribution could be considered. The performance of the one-factor model used could be improved by using a dynamic factor latent model. In this case, the correlation of asset returns is not constant as in the case of the static factor model. This type of model leads to very complicated likelihood functions. More advanced numerical techniques are necessary for their estimation. Elaboration of the stress scenario could be used to analyze the influence on default rates in the Finnish economy.

Although the Finnish economy was hit by a strong recession and structural changes in the early 1990s, the performance of the estimated model was fairly good. Our study proved a significant influence of macroeconomic variables on default rates in the economy. Differences between industrial sectors were shown. Our study investigated and compared two possible approaches to credit risk modeling. The latent factor model was found to be more powerful in macroeconomic modeling of default rates. We estimated the one-factor model for the aggregate economy and also for industry-specific sectors. These models can be used for stress testing or default rate prediction.

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Appendix

TABLE 23 Order of stationarity of default rates

Name of Variable	Short Name of Variable	Order of Stationarity
Default Rate	$\mathtt{df}$	I(1)
Default Rate in Agriculture	$\mathtt{df}_{\mathtt{AGR}}$	I(0)
Default Rate in Construction	$\mathtt{df}_{\mathtt{CON}}$	I(1)
Default Rate in Manufacturing	$\mathtt{df}_{\mathtt{MAN}}$	I(1)
Default Rate in Trade	$\mathtt{df}_{\mathtt{TRD}}$	I(1)
Default Rate in Transport	$\mathtt{df}_{\mathtt{TRN}}$	I(1)

TABLE 24 Order of stationarity of macroeconomic indicators

Name of Variable	Short Variable Name	Order of Stationarity
Real GDP deviation	dGDP	I(1)
Real GDP difference from trend	GDPdif	I(1)
Nominal interest rate 1M	r1M	I(1)
Nominal interest rate 3M	r3M	I(1)
Nominal interest rate 12M	r12M	I(1)
Real interest rate 1M (CPI)	$\mathtt{r1M}_{\mathtt{CPI}}$	I(1)
Real interest rate 3M (CPI)	$\mathtt{r3M}_{\mathtt{CPI}}$	I(1)
Real interest rate 12M (CPI)	$\mathtt{r12M}_{\mathtt{CPI}}$	I(1)
Real interest rate 1M (PPI)	$\mathtt{r1M}_{\mathtt{PPI}}$	I(1)
Real interest rate 3M (PPI)	$\mathtt{r3M}_{\mathtt{PPI}}$	I(1)
Real interest rate 12M (PPI)	$\mathtt{r12M}_{\mathtt{PPI}}$	I(1)

TABLE 25 Order of stationarity of value added

Name of Variable	Short Name of Variable	Order of Stationarity
difference of real value added in agriculture	$\mathtt{dGDP}_{\mathtt{AGR}}$	I(0)
difference of real value added in construction	$\mathtt{dGDP}_{\mathtt{CON}}$	I(1)
difference of real value added in manufacturing	$\mathtt{dGDP}_{\mathtt{MAN}}$	I(1)
difference of real value added in trade	$\mathtt{dGDP}_{\mathtt{TRD}}$	I(1)
difference of real value added in transport	$\mathtt{dGDP}_{\mathtt{TRN}}$	I(1)