

LUKÁŠ VÁCHA
MILOSLAV VOŠVRDA***HETEROGENEOUS AGENTS MODEL
WITH THE WORST OUT ALGORITHM****Abstract:**

Heterogeneous agents' model with the stochastic beliefs formation is considered. Fundamentalists rely on their model employing fundamental information basis to forecast the next price period. Chartists determine whether current conditions call for the acquisition of fundamental information in a forward looking manner rather than relying on the past performance. It was shown that implementation of the agents memory can significantly change the preferences of trader strategies. The Worst out Algorithm (WOA) is used with considered heterogeneous agents' model to simulate more realistic market conditions. The WOA replaces periodically the trading strategy that has the lowest performance level of all strategies presented on the market by the new one. The memory length of the new strategy that enters the market has the same stochastic structure as the initial strategies. This paper shows an influence of the agent memory as a stochastic process on the heterogeneous agents model with the WOA. Simulations show difference in price returns behaviour between two types of agents' memory length distribution functions (Uniform and Normal). There is a significant difference in the values of the Hurst exponent and the variance in these two cases. A lower Hurst exponent in the uniform case is caused by a richer spectrum of agents' memory length, because agents are equally distributed across all trading horizons. For the uniform case there is no opportunity for any prediction. On the other hand, the value of the Hurst exponent gives a signal for a possibility of the price prediction in the normal case.

Keywords: Efficient Markets Hypothesis, Fractal Market Hypothesis, agents' investment horizons, agents' trading strategies, technical trading rules, heterogeneous agent model with stochastic memory, Worst out Algorithm

JEL Classification: C61, G14, D84

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1. Introduction

Assumptions about rational behaviour of agents, homogeneous models, and efficient market hypothesis were paradigms of economic and finance theory for the last years. After empirical data analysis on financial markets and economic and finance progress these paradigms are gotten over. There are phenomena observed in real data collected from financial markets that cannot be explained by the recent economic and finance theories. Introducing non-linearity in

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the models may improve research of a mechanism generating the observed movements in the real financial data. Financial markets are considered as systems of the interacting agents processing immediately new information. Heterogeneity in expectations can lead to market instability and complicated dynamics. Our approach assumes that agents are intelligent ones having no full knowledge about the underlying model in the sense of the rational expectation theory and have not the computational equipment to interpret obtained information by different ways. Prices are driven by endogenous market forces. Adaptive belief approach is employed – see (Brock, Hommes, 1997). Agents adapt their predictions by choosing among a finite number of predictors. Each predictor has a performance (efficient) measure. Based on this performance, agents realize a rational choice among the predictors. Brock and Hommes showed that the adaptive rational equilibrium dynamics incorporates a general mechanism, which may generate local instability of the equilibrium steady state and complicated global equilibrium dynamics – see (Brock, Hommes, 1997). Vosvrda and Vacha focused on a version of the model with two types of trades, i.e., fundamentalists, and technical traders (see (Vosvrda, Vacha, 2002a), (Vosvrda, Vacha, 2003)). This approach relied on heterogeneity in the agent information and subsequent decisions either as fundamentalists or as chartists. A more detailed analysis is introduced in the Brock and Hommes model (Brock, Hommes, 1997). The model with memory was analysed in (Vosvrda, Vacha, 2002b). A process of a memory feeding is improved by the worst out algorithm (WOA).

Let us consider an asset-pricing model with one risky asset and one risk-free asset. Let p_t be the share price (ex dividend) of the risky asset at time t , and let $\{y_t\}$ be an i.i.d. stochastic dividend process of the risky asset. The risk free asset is perfectly elastically supplied and pays a fixed rate of return r . The gross return r^g is equal $1+r$. The risky asset pays a random dividend. The dynamics of wealth can then be written as

$$\mathbf{W}_{t+1} = r^g \cdot \mathbf{W}_t + (\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t) \cdot z_t \quad (1.1)$$

where z_t denotes the number of shares of the asset purchased at time t , and a bold face type denotes random variables at date t . Let E_t and V_t denote the conditional expectation and conditional variance operators, based on the publicly available information set consisting of past prices and dividends, i.e., on the information set $\mathfrak{S}_t = \{p_b, p_{t-1}, \dots; y_b, y_{t-1}, \dots\}$. Let $E_{h,t}$, $V_{h,t}$ denote forecasts of investor of type h about a conditional expectation and conditional variance. Investors are supposed to be a myopic mean-variance maximizer so that the demand $z_{h,t}$ for risky asset is obtained by solving the following criterion

$$\max_{z_t} \left\{ E_{h,t} [\mathbf{W}_{t+1}] - (a/2) \cdot V_{h,t} [\mathbf{W}_{t+1}] \right\} \quad (1.2)$$

where a risk aversion, a , is here assumed to be the same for all traders. Thus the demand $z_{h,t}$ of type h for risky asset has the following form

$$\begin{aligned} E_{h,t} [\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t] - a \cdot \sigma^2 \cdot z_{h,t} &= 0 \\ z_{h,t} &= E_{h,t} [\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t] / (a \cdot \sigma^2) \end{aligned} \quad (1.3)$$

assuming that the conditional variance of excess returns is a constant for all investor types

$$V_{h,t} (\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t) \equiv \sigma_h^2 = \sigma^2 \quad (1.4)$$

Let Z^s be a supply of outside risky shares per investor. Let $n_{h,t}$ be a fraction of type h at date t . The equilibrium of demand and supply is

$$\sum_{h=1}^H n_{h,t} \left\{ E_{h,t} \left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t \right] / a \cdot \sigma^2 \right\} = z^s \quad (1.5)$$

where H is the number of different investor types. For the special case of zero supply, i.e., $z^s = 0$, the market equilibrium is as follows

$$r^g \cdot p_t = \sum_{h=1}^H n_{h,t} \cdot E_{h,t} \left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} \right]$$

If there is only one investor type, the market equilibrium yields the following pricing equation

$$r^g \cdot p_t = E_t \left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} \right] \quad (1.6)$$

It is well known that, using the arbitrage (1.6) repeatedly and assuming that the transversality condition

$$\lim_{k \rightarrow \infty} \frac{E_t \left[\mathbf{p}_{t+k} \right]}{\left(r^g \right)^k} = 0 \quad (1.7)$$

holds, the fundamental price of the risky asset is uniquely obtained by

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t \left[\mathbf{y}_{t+k} \right]}{\left(r^g \right)^k} \quad (1.8)$$

Thus the fundamental price p_t^* depends on the stochastic dividend process $\{y_t\}$. From the equation (1.6) we obtain the following price equation

$$p_t = p_t^* + \left(r^g \right)^t \cdot \left(p_0 - p_0^* \right) \quad (1.9)$$

For our purpose, it is better to work with the deviation x_t from the benchmark fundamental price p_t^* , i.e., $x_t = p_t - p_t^*$.

2. Evolutionary Dynamics of Investors

Let us admit the following assumptions:

A1)

$$E_{h,t} \left[\mathbf{y}_{t+1} \right] = E_t \left[\mathbf{y}_{t+1} \right] \quad (2.1)$$

A2)

$$V_{h,t} \left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t \right) = V_t \left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t \right) = \sigma_t^2 \quad (2.2)$$

A3) All forecasts $E_{h,t} \left[\mathbf{p}_{t+1} \right]$ have the following form

$$E_{h,t} \left[\mathbf{p}_{t+1} \right] = E_t \left[\mathbf{p}_{t+1}^* \right] + f_h^{\mathbf{L}} \left(x_{t-1}, \dots, x_{t-\mathbf{L}} \right) \quad (2.3)$$

Each forecast $f_h^{\mathbf{L}}$ represents a model of the market for which type h believes that prices deviate from the fundamental price. Let us concentrate on the evolutionary dynamics of the fractions $n_{h,t}$ of different h -investor types, i.e.

$$r^g \cdot x_t = \sum_{h=1}^H n_{h,t-1} \cdot f_h^{\mathbf{L}}(x_{t-1}, \dots, x_{t-\mathbf{L}}) \equiv \sum_{h=1}^H n_{h,t-1} f_{h,t}^{\mathbf{L}} \quad (2.4)$$

where $n_{h,t-1}$ denotes the fraction of investor type h at the beginning of period t , before than the equilibrium price x_t has been observed and \mathbf{L} is a random number of lags. Now the realized excess return over period t to the period $t+1$ is computed by

$$\mathbf{Z}_{t+1} = \mathbf{p}_{t+1} + \mathbf{y}_{t+1} - r^g \cdot p_t \quad (2.5)$$

$$\mathbf{Z}_{t+1} = \mathbf{x}_{t+1} + \mathbf{p}_{t+1}^* + \mathbf{y}_{t+1} - r^g \cdot x_t - r^g \cdot p_t^* \quad (2.6)$$

$$\mathbf{Z}_{t+1} = \mathbf{x}_{t+1} - r^g \cdot x_t + \mathbf{p}_{t+1}^* + \mathbf{y}_{t+1} - E_t[\mathbf{p}_{t+1}^* + \mathbf{y}_{t+1}] + E_t[\mathbf{p}_{t+1}^* + \mathbf{y}_{t+1}] - r \quad (2.7)$$

From the equations (1.3), and (2.4), we get the following expressions

$$E_t[\mathbf{p}_{t+1}^* + \mathbf{y}_{t+1}] - r^g \cdot p_t^* = 0, \mathbf{p}_{t+1}^* + \mathbf{y}_{t+1} - r^g \cdot p_t^*, \text{ and} \quad (2.8)$$

$$\mathbf{x}_{t+1} - r^g \cdot x_t$$

The process $\{\mathbf{x}_{t+1} - r^g \cdot x_t\}$ is a martingale difference sequence with respect to \mathfrak{F}_t under

EMH, i.e., let us put $\delta_{t+1} = \mathbf{x}_{t+1} - r^g \cdot x_t$ and thus $E_t[\delta_{t+1}] = (1 - r^g) \cdot x_t$ for all t . So

Eq. (2.7) can be written as follows

$$E_t[\mathbf{Z}_{t+1}] = E_t[\mathbf{p}_{t+1}^* + \mathbf{y}_{t+1} - r^g \cdot p_t^*] + E_t[\delta_{t+1}] = (1 - r^g) \cdot x_t. \quad (2.9)$$

The decomposition of the equation (2.9) as separating the ‘explanation’ part of realized excess returns \mathbf{Z}_{t+1} into the contribution $\mathbf{p}_{t+1}^* + \mathbf{y}_{t+1} - r^g \cdot p_t^*$ and the additional part δ_{t+1} . We need now a measure of profits generated by forecasts $f_{h,t}^{\mathbf{L}}$. Let a performance measure $\pi_{h,t}$ be defined by

$$\pi_{h,t} = E_t \left[\frac{\mathbf{Z}_{t+1} \cdot \rho_{h,t}}{a \cdot \sigma_t^2} \right] \quad (2.10)$$

where

$$\rho_{h,t} = E_{h,t}[\mathbf{Z}_{t+1}] = f_{h,t}^{\mathbf{L}} - r^g \cdot x_t = f_{h,t}^{\mathbf{L}} - \sum_{j=1}^H n_{j,t} \cdot f_{j,t}^{\mathbf{L}} = f_{h,t}^{\mathbf{L}} \cdot \left(1 - \sum_{j \neq h} n_{j,t} \cdot f_{j,t}^{\mathbf{L}} \right). \quad (2.11)$$

So the π -performance is given by the realized performance for the h -investor. Let the updated fractions $n_{h,t}$ be given by the discrete choice probability (Gibb’s distribution)

$$n_{h,t} = \exp(\beta \cdot \pi_{h,t-1}) / Y_{t-1} \quad (2.12)$$

$$Y_t = \sum_{j=1}^H \exp(\beta \cdot \pi_{j,t}) \quad (2.13)$$

The parameter β is the intensity of choice measuring the amount of uncertainty in choice. We can say the more uncertainty the lesser the parameter β . The parameter β is a measure of investor's rationality. If the intensity of choice is infinite ($\beta = +\infty$), the entire mass of investors uses the strategy that has the highest performance. If the intensity of choice is zero, the mass of investors distributes itself evenly across the set of available strategies. All forecasts will have the following form

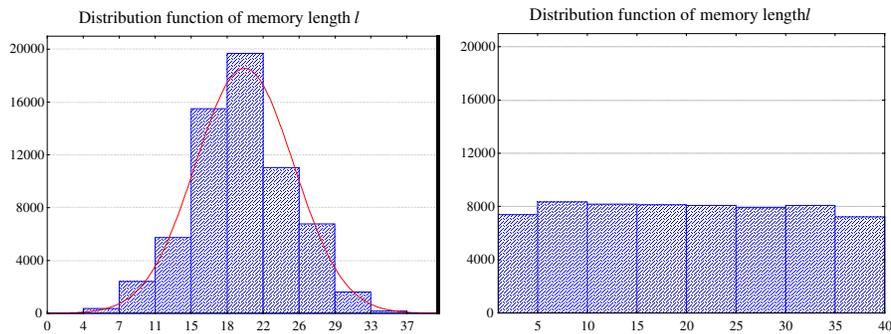
$$f_t^L = g \cdot (x_{t-1} + \dots + x_{t-l}) + b \quad (2.14)$$

where the g denotes the trend of investor, and the b denotes the bias of investor. If $b = 0$, the investor is called a pure trend chaser if $g > 0$ and a contrarian if $g < 0$. If $g = 0$, investor is called purely biased. Investor is upward (downward) biased if $b > 0$ ($b < 0$). In the special case $g = b = 0$, the investor is called fundamentalist, i.e., the investor believes that price return to their fundamental value. Fundamentalists strategy is based on all past prices and dividends in their information set, but they do not know the fractions $n_{h,t}$ of the other belief types.

3. Monte Carlo Simulations of the Financial Market Agents

The main idea of this paper is a comparison of behaviour of two cases, which differs in the distribution function $F(l)$ which controls memory length l of the agents on the financial market. The first case is with the normal memory length distribution, $F_N(l) \sim N(20,25)$, the second case has the uniform memory length distribution $F_U(l) \sim U(1,40)$, see Figure 1. Parameters g (trend) and b (bias) have the same stochastic structure for the two cases i.e., $N(0,0.16)$ and $N(0,0.09)$, respectively (see the equation (2.14)). These distributions are used for forming the initial set of agents' beliefs (trading strategies) and the same distribution functions are also used for adding a new agent, using the WOA. Here, a memory length is considered as a stochastic vector. The length of the memory is a number of stochastic elements in the stochastic vector. The performance measure is computed as a moving average with length l , where l represents memory length of a particular agent on the market. The moving average has equal weights, so there is no memory fading present in the memory process.

FIGURE 1 Distribution Functions of Memory Length, left: uniform, right: normal



Simulations show difference between these two types of agents' memory distribution functions. Simulations are performed with A agents (beliefs), where $A = 40$, with the intensity of choice $\beta = 120$. The WOA algorithm is set to enter the process after every 50 iterations. Number of observations is 10 600 for both cases.

3.1 Hurst Exponent

For estimating and analyzing of correlation structures on capital markets, a non-parametric method is used. H. E. Hurst discovered very robust nonparametric methodology which is called rescaled range or R/S analysis which is used for estimating the Hurst exponent Peters (1994). The R/S analysis was used for distinguishing random and non-random systems, the persistence of trends, and duration of cycles. This method is very convenient for distinguishing random time series from fractal time series as well. Starting point for the Hurst's coefficient was the Brownian motion as a primary model for random walk processes.

If a system of random variables is an i.i.d., then $H = 0.5$, i.e., Geometrical Brownian Motion (GBM) that is shown as a dashed line in figures with R/S analysis. The values of Hurst exponent belonging to $0 < H < 0.5$ signifies antipersistent system of variables covering less space than random ones. Such a system must reverse itself more frequently than a random process, we can equate this behavior to a mean-reverting process. Values $0.5 < H < 1$ show persistent process that is characterized by long memory effects. This long memory occurs regardless of time scale, i.e., there is no characteristic time scale which is the key characteristic of fractal time series (Peters, 1994).

FIGURE 2 Normal Distributed of the Memory Length, $H = 0.519$ (10 600 observations)

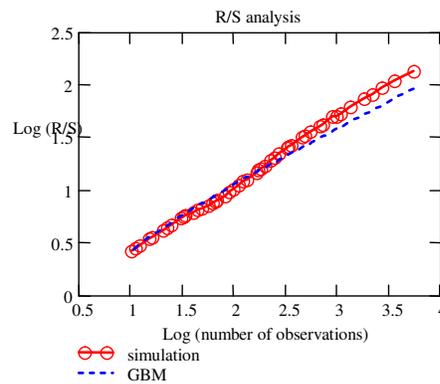


Figure 2¹ shows results of the R/S analysis for the first case, where the memory length distribution function is normal $F_N(l)$ (the normal case). The whole time series (10 600 observations) was analyzed. According to the estimate of the Hurst exponent the time series of asset price returns is slightly persistent ($H = 0.519$). This is in contrast to the second case where the memory length distribution function is uniform $F_U(l)$ (the uniform case), see in Figure 3, where the time series of asset price returns is antipersistent ($H = 0.412$). The results of these simulations indicate that there is a possibility of a prediction in the normal case – see (Peters, 1994).

The values of the Hurst exponent are not the same during all simulation. The WOA changes these values significantly and we can observe distinct trend of the Hurst exponent in the two cases. We estimated the Hurst exponent in a set of the first and the last 3600 obser-

¹ The solid line represents the output of the model (average of many simulations), the dashed line represents the GBM case i.e., an i.i.d. process.

variations, see *Figures 4*, and *5*. In the normal case, the time series of asset price return becomes more persistent as more changes with the WOA are done i.e., from $H = 0.423$ to $H = 0.589$, *Figure 4*, conversely, in the uniform case the time series becomes more antipersistent i.e., from $H = 0.369$ to $H = 0.286$, see *Figure 5*. Descriptive statistics of the examined asset price returns time series are in *Table 1*, there is noticeable difference in the values of kurtosis. The normal case has remarkably higher kurtosis than the uniform case, and in fact, is closer to the real financial markets.

FIGURE 3 Uniform Distributed of the MemoryLength, $H = 0.412$ (10 600 observations)

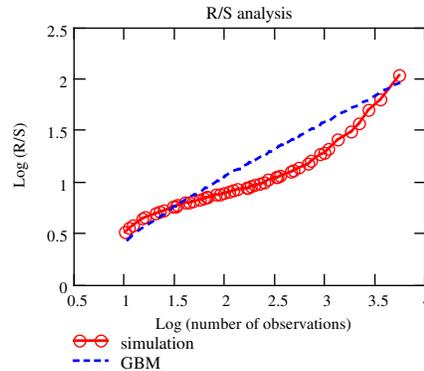
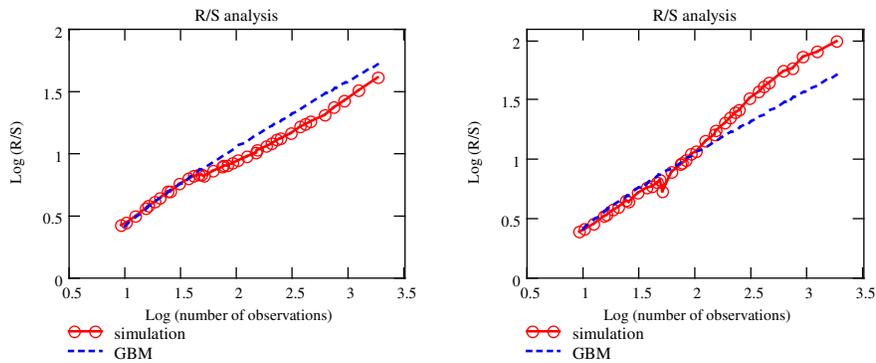


FIGURE 4 Normal Memory Length Distribution, *left*: the first 3600 obs., $H = 0.423$, *right*: the last 3600 obs. (7000–10600), $H = 0.589$



3.2 Impact of the WOA on Trading Strategies on the Financial Market

For a better understanding of the evolution dynamics with the WOA we compared market structure i.e., what types of agents (beliefs) are presented on the market, at the beginning and at the end of the experiment, *Figures 6–9*. We were interested in how the set of strategies on market is changed after several replacements under the WOA i.e., what types of strategies survive on the market. We can see the contrast between the examined cases with different memory distribution function, which is the only distinction between these cases.

FIGURE 5 Uniform Memory Length Distribution, *left*: the first 3600 obs., $H = 0.369$, *right*: the last 3600 obs. (7000–10 600), $H = 0.286$

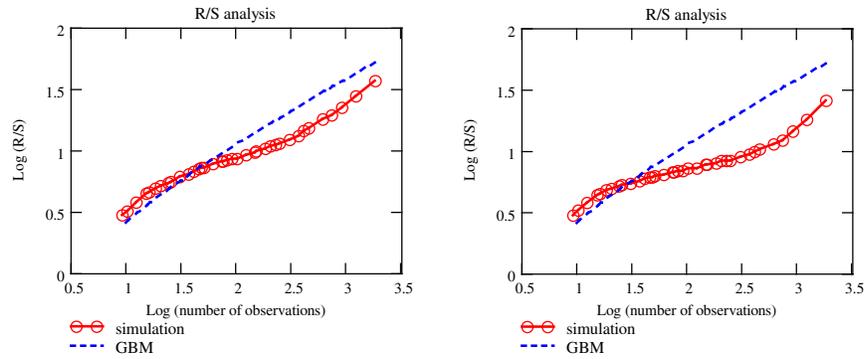
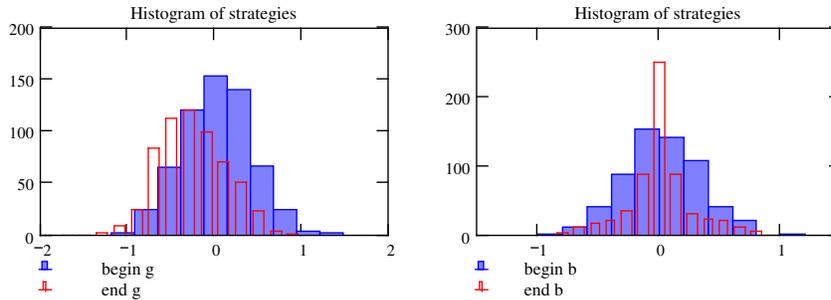


TABLE 1 Descriptive Statistics and Hurst Exponent for the Normal and the Uniform Case

Case	Hurst e.	Mean	Variance	s.d.	Kurtosis	Skewness
Normal (0–10 600)	0.519	0.0000	0.029	0.17	5.508	-0.149
Normal (0–3600)	0.423	0.0005	0.033	0.18	3.189	-0.129
Normal (7000–10 600)	0.589	-0.0005	0.026	0.16	2.448	-0.236
Uniform (0–10 600)	0.412	-0.0095	0.102	0.32	0.584	0.092
Uniform (0–3600)	0.369	-0.0130	0.076	0.28	0.144	0.032
Uniform (7000–10 600)	0.286	0.0020	0.134	0.37	-0.591	0.011

FIGURE 6 Initial and Final Distribution of Agents' Strategies for the Normal Case, *left*: trend g , *right*: bias b (10 600 observations)



3.2.1 Normal Case

For the normal case, there is an evident variance decrease (very strong in some experiments, see Figure 13). This noticeable fact is caused by market learning that means presence of the WOA that eliminates unsuccessful trading strategies. The impact of the market learning is depicted in histograms (Figures 6, 7), where the shift of trading strategies preferences is clear. A solid bar describes the initial empirical distribution of agents' strategies (trend g , bias b , memo-

ry l); the empty bar is the final empirical distribution after all changes under the WOA are done. Figure 6 shows shift to the contrarians' strategies (lower mean for the end g , Table 2) and a central tendency to the zero bias (higher kurtosis for the end b , Table 2). The mean of the memory for the normal case is lower in the final set of trading strategies, which signalize the problem of the traders with longer trading horizons to react to price fluctuations on the market.

FIGURE 7 Initial and Final Distribution of Agents' Memory for the Normal Case (10 600 observations)

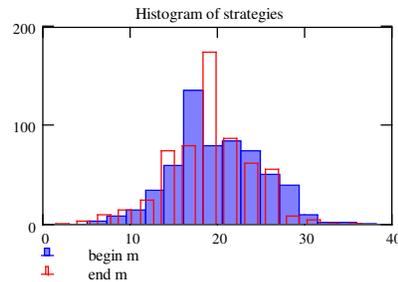
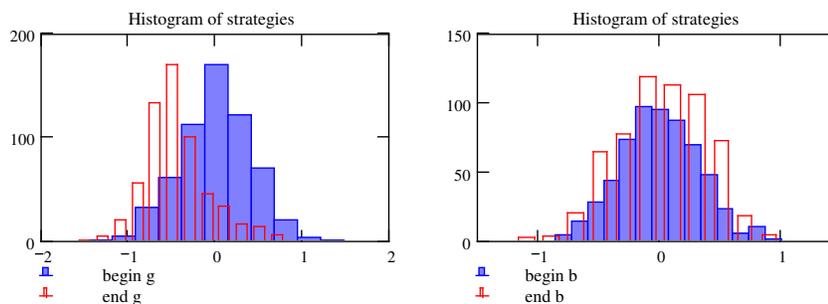


TABLE 2 Descriptive Statistics of the Initial and Final Trading Strategies, the Normal Case

Normal case	Mean	Variance	s.d.	Kurtosis	Skewness
Begin g	0.0120	0.153	0.40	-0.1890	0.093
End g	-0.2620	0.141	0.36	-0.3650	0.200
Begin b	0.0200	0.092	0.30	-0.1310	0.089
End b	0.0058	0.063	0.25	1.6860	0.083
Begin m	19.7880	24.624	4.96	-0.0190	0.073
End m	18.9830	22.460	4.74	0.0539	-0.207

FIGURE 8 Initial and Final Distribution of Agents' Strategies for the Uniform Case
left: trend g , right: bias b (10 600 observations)



3.2.2 Uniform Case

The uniform case reveals a strong shift of a trend preference mean to contrarians, more than one σ , in Figure 8, left, and Table 3, that is stronger than in the normal case. Parameter bias b has practically the same descriptive statistics during the simulation. Alike in the normal case,

a part of traders with longer memory is eliminated by the WOA, this situation is depicted in *Figure 9*, where more than a double increase in the first column of the histogram occurs, i.e., the evolutionary dynamics on the financial market is more favorable to traders with shorter memory length. Due to the increase of trading strategies with very short memory length, we can observe increasing values of asset price returns variance in time, see *Table 1*, and *Figure 11*.

FIGURE 9 Initial and Final Distribution of Agents' Memory for the Uniform Case (10 600 observations)

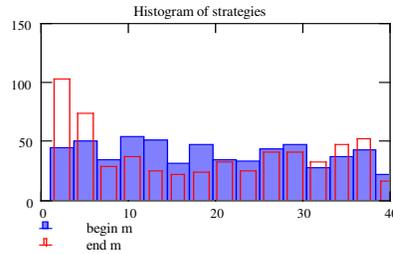


TABLE 3 Descriptive Statistics of the Initial and Final Trading Strategies, the Uniform Case

Uniform case	Mean	Variance	s.d.	Kurtosis	Skewness
Begin <i>g</i>	-0,0160	0.164	0.400	-0.0084	-0.101
End <i>g</i>	-0.4520	0.129	0.360	0.9650	0.732
Begin <i>b</i>	-0.0048	0.099	0.314	-0.1810	-0.074
End <i>b</i>	0.0150	0.128	0.357	-0.4720	-0.176
Begin <i>m</i>	19.483	128.62	11.340	-1.1930	0.134
End <i>m</i>	18.213	164.351	12.810	-1.4680	0.131

3.3 Analysis of Variance

This subsection deals with the evolution of an average of variances of all fifteen experiments of the two cases we are discussing above, i.e., the cases with the normal and the uniform distribution function of the memory length. The mean of memory length is almost equal in these two cases to eliminate the occurrence of difference in the variance values due to substantial disparity of the memory lengths. In (Vavra, Vosvrda, 2005) is proved that the higher the mean of the memory length the lower the variance of the asset price returns.

Behaviour of the variance is considerably different in the two cases. In general, the normal case has lower variance in almost all experiments, this situations is depicted in histogram of all experiments' variances, *Figure 10*. *Figure 11* shows a trend of variance. For the variance development in time, we use moving a window of length 100. In the uniform case there is a linear trend of the variance. Conversely, in the normal case, there is a slow variance decrease. We can say that the WOA causes a risk stabilizing role in the normal case. In the uniform case, the WOA causes the increase of the market risk level.

Analyzing the variance of the asset price returns in the normal case more closely we have found two groups of realizations with different behaviour of variance in time, see *Figure 12*. We can see that the variance evolution of these groups is dissimilar. The group with the lowest variance (four experiments), in *Figure 12* left panel, has a quickly decreasing variance, that after 4000 iterations falls to very small levels close to zero, on the other hand the group with the highest variance, right panel, has a constant zero trend after 4000 iterations. This result

indicates that the initial mix of trading strategies in the case with the uniform memory length distribution function, *Figure 13*, does not play such a big role as in the case with the normal memory length.

FIGURE 10 Histogram of Averages of Total Variances for the Cases with the Uniform and the Normal Memory Distribution Function

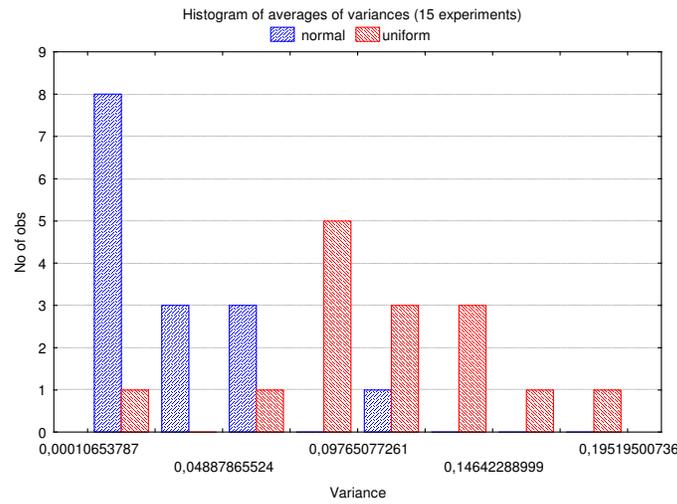
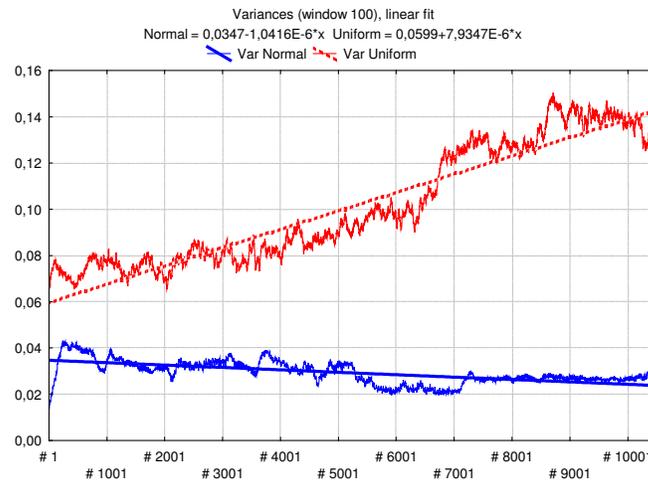


FIGURE 11 Average of Variances (with Linear Fit) for the Cases with the Uniform and the Normal Memory Distribution Function



4. Conclusion

The important outcome of the simulations is the possibility of prediction in the case with the normally distributed memory length. An interesting result concerning risk behavior is the fact that the WOA plays a stabilizing role in the normal case in a sense of decreasing variance in

FIGURE 12 Variance for the Case with the Normal Memory Distribution Function
left: 4 experiments with the lowest variance,
right: 4 experiments with the highest variance

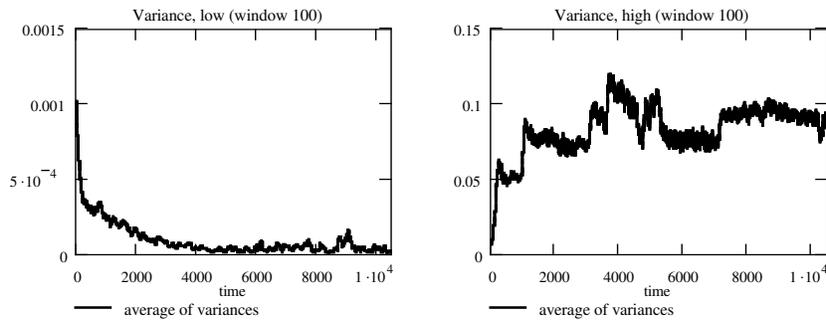
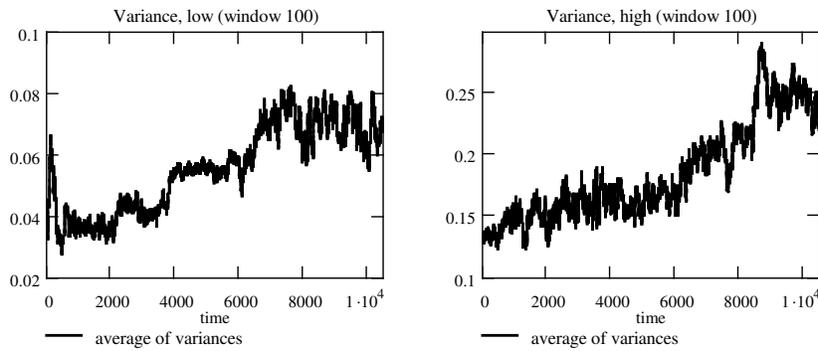


FIGURE 13 Variance for the Case with the Uniform Memory Distribution Function
left: 4 experiments with the lowest variance,
right: 4 experiments with the highest variance



time. Conversely, the uniform case affects the financial market risk level negatively, i.e., rising variance in time. Another interesting result is the fact that in the uniform case there is a strong shift of a trend preference mean to contrarians, stronger than in the normal case. It means that with the WOA contrarians are more effective agents than trend chasers agents.

5. References

- Brock WA (2001): *Growth Theory, Nonlinear Dynamics and Economic Modelling*. Edward Elgar.
- Brock WA, Hommes CH (1997): A rational route to randomness. *Econometrica*, 65:1059–1095.
- Brock WA, Hommes CH (1998): Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic Dynamics and Control*, 22:1235–1274.
- Chiarella C (1992): The dynamics of speculative behaviour. *Annals of Operational Research*, 37: 101–123.
- Chiarella C, He X (2000): Heterogeneous beliefs, risk and learning in a simple asset pricing model with a market maker. Quantitative Finance Research Group. University of Technology Sydney, *Research Paper*, no. 35.
- Gaunersdorfer A (2000): Endogenous fluctuations in a simple asset pricing model with heterogeneous agents. *Journal of Economic Dynamics and Control*, 24: 799–831.
- Lucas RE (1978): Asset prices in an exchange economy. *Econometrica*, 46:1429–1445.
- Haltiwanger J, Waldman M (1985): Rational expectations and the limits of rationality: an analysis of heterogeneity. *American Economic Review*, 75(3):326–340.
- Lorenz H-W (1993): *Nonlinear Dynamical Economics and Chaotic Motion*. Springer-Verlag, Berlin.
- Lux T (1997): *Economics Dynamics. Phase diagrams and their economic application*. Cambridge. Cambridge University Press.
- Peters EE (1994): *Fractal Market Analysis, Applying Chaos Theory to Investment and Economics*. Springer.
- Vosvrda M (2001): *Bifurcation Routes in Financial Markets*. 19th International Conference Mathematical Methods in Economics 2001, Hradec Kralove.
- Vosvrda M, Vacha L (2002a): Heterogeneous Agent Model with Memory and Asset Price Behaviour. *Proceedings of the 20th International Conference Mathematical Methods in Economics 2002, Ostrava*, 273–282.
- Vosvrda M, Vacha L (2002b): Heterogeneous Agent Model and Numerical Analysis of Learning. *Bulletin of the Czech Econometric Society*, 17:15–22
- Vosvrda M, Vacha L (2003): Heterogeneous Agent Model with Memory and Asset Price Behaviour. *Prague Economic Papers*, 12(2):155–168.
- Vacha L, Vosvrda M (2005): Dynamical Agents' Strategies and the Fractal Market Hypothesis. *Prague Economic Papers*, 14(2):172–179.