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Education, Endogenous Human Capital, and Monetary Economic Growth with MIU Approach

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Abstract This study builds a monetary growth model with inflation policy and education. The model is a synthesis of the Uzawa-Lucas two-sector growth model and traditional monetary model with the money-in-utility (MIU) approach. We show how money, physical capital and human capital interact over time under exogenous inflation policy in a free market economy. The dynamics of the economy is described by three differential equations. We show that the monetary economic system has a saddle equilibrium point. We simulate motion of the economic system and carry out comparative statics analysis with regards to the monetary policy, propensities to save wealth and to receive education.

Keywords Economic growth, MIU approach, inflation policy, physical capital, education and human capital, learning through consuming
JEL classification I25, O41, E50

1. Introduction

The effects of education and monetary policy on economic growth and development have been long-standing issues in economics. There are many theoretical and empirical studies on money and growth or education and economic growth. But there are few formal economic models which deal with monetary policy, education, and economic growth and development in a single consistent framework. The purpose of this study is to synthesize the main economic mechanisms in the two important models in the literature of economic theory to examine interactions among monetary changes, physical capital accumulation, and human capital accumulation with inflation policy and education.

The seminal contribution in the theory of monetary growth was published by Tobin (1965). Tobin studies an isolated economy in which outside money competes with real capital in the portfolios of agents within the framework of the Solow model. In the Tobin model there is also a real sector exactly like that in the Solow growth model. The Tobin model lacks of the microeconomic foundation for determining money holding.

An important approach in monetary growth economics with microeconomic foundation is the so-called the utility (MIU) function approach. In this approach money is held because it yields some services and the way to model it is to enter real balances

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directly into the utility function.[1] The approach was used initially by Patinkin (1965), Sidrauski (1967a) and Friedman (1969) and has been applied to different issues related to monetary phenomena. In his well-known paper, Sidrauski (1967b) challenged Tobin's non-neutrality result. He found that money is superneutral in steady state and changes in the inflation rate have no effect on all the real variables in the economy.[2] Nevertheless, it has become evident that his results are dependent on the specific set-up of the model. For instance, the superneutrality in Sidrauski's model is no more held if leisure is introduced into the utility function. Wang and Yip (1992) show that the direction of the non-superneutrality result is related to the signs of the cross-partial derivatives of the utility function with respect to consumption, leisure, and real balances.[3]

Another model is developed by Meng and Yip (2004), who add endogenous investment to a flexible-price to the conventional monetary growth model. Their analytical results show that physical capital is significant for stabilizing the real side of the economy when the monetary authority follows interest-rate feedback rules. They examine behavior of the model both when labor supply is inelastic and is endogenous either with active monetary policy (which responds to one percentage point increase in inflation with a more than one percentage point increase in the nominal interest rate) or passive monetary policy (which responds to a one percentage point increase in inflation with a less than one percentage point increase in the nominal interest rate).[4]

This study is strongly influenced by the traditional monetary growth model in the MIU approach. But we deviate from the literature in that we use an alternative utility function to modeling household behavior and we introduce education and human capital into the monetary growth model.

Human capital and education are important for modern economic growth (for instance, Hanushek and Kimko 2000; Castelló-Climent and Hidalgo-Cabrillana 2012; and Brodzicki 2012). There are two main issues in the research of education and economic growth. The first one is individual returns to education. The second one is the relation between education and economic growth.

Estimating the return to education has caused great attention in empirical studies in economics since Mincer (1974) published the seminal work in 1974. For instance, Fleisher et al. (2011) study the role of education on worker productivity and firms' total factor productivity on the basis of firm-level data from China. The study shows that an additional year of schooling raises marginal product by 30.1 percent, and the CEO's education increases TFP for foreign-invested firms. Another recent study by Li et al. (2012) shows that the high school education in China has low returns in terms of earnings and may mainly serve as a mechanism to select students for higher education;

[1] See Eden (2005, chap. 2) for the reasons why money is introduced into the utility function.

[2] Superneutrality of money means that the growth rate of money has no effect on the real equilibrium.

[3] It should be noted that Feenstra (1986) shows that the MIU approach is functionally equivalent to the transactions-cost model. It has also been shown that either the shopping-time model or cash-in-advance model can be rewritten as a MIU model.

[4] There are many other issues and models related to interactions between monetary policy and economic growth, for instance, Gomme (1993), van der Ploeg and Alogoskoufits (1994), Jones and Manuelli (1995), Dotsey and Starte (2000), Chappell and Matthews (2001), Chang et al. (2007), Nelson (2008) and Handa (2009).

both vocational school education and college education have a large return which is comparable to that found in the U.S.

In the literature of economic growth there are many models about interdependence between education and economic growth. The first formal dynamic growth model with education and microeconomic foundation was proposed by Uzawa (1965). In the Uzawa model there are industrial and education sectors. As far as growth and education are concerned, the work by Lucas (1988) is similar to Uzawa's model in many aspects. Lucas's work has caused a great interest among economists in issues related to relations between growth and human capital. The Uzawa-Lucas model has been extended and generalized in various directions (Zhang 2005). This study makes another contribution to the literature by introducing money into the Uzawa-Lucas model.

The MIU approach is a key modeling framework in the literature of monetary growth theory, while the Uzawa-Lucas MIU approach is a key modeling framework in the literature of growth with endogenous physical and human capital accumulation. Nevertheless, the two key approaches in the literature of economic theory have been separated. It is reasonable to synthesize these two separate approaches within a single framework. This study is a synthesis of a model of education and growth by Zhang (2007) and a model of monetary growth by Zhang (2010). Zhang's 2007 model, which is an extension of the Uzawa-Lucas two sector model, does not include monetary variables, while Zhang's 2010 model, which is influenced by the growth model with the MIU approach, does not consider education. This study synthesizes the main ideas in Zhang's two models.

The paper is organized as follows. Section 2 defines the monetary growth model with physical capital and human capital accumulation. Section 3 shows that the economy is described by three differential equations and also simulates the model. Section 4 carries out comparative statics analysis with regards some parameters. Section 5 concludes the study.

2. The monetary growth model with education

Like the Uzawa-Lucas model, the economic system in our study consists of the industrial and education sectors. The production aspects of the economic system are similar to the Solow one-sector neoclassical growth model (Zhang 2005), except that saving is endogenous in our model. Production is generally described as combination of multiple production factors such as labor and capital. Time is represented continuously by a numerical variable which takes on all values from zero onwards. Let $T(t)$ stand for the work time of a representative household and $N(t)$ for the flow of qualified labor services used at time t for production. We measure $N(t)$ as follows

$$N(t) = T(t)\, H^{\omega}(t)\, \overline{N} \tag{1}$$

where $H(t)$ is the level of human capital and is $\overline{N}$ the population Here, ω is a positive parameter measuring how the worker effectively applies human capital.

We use the conventional production function to describe a relationship between inputs and output. Total capital, $K(t)$ is fully used by the two sectors. We denote

$K_i(t)$ and $K_e(t)$, respectively, the capital stocks employed by the industrial sector and education sector. We have

$$K(t) = K_i(t) + K_e(t). \tag{2}$$

Let $\bar{k}(t)$ stand for the value of capital owned per household. We have $\bar{k}(t) = K(t)/\bar{N}$. We use $N_i(t)$ and $N_e(t)$ to stand for the qualified labor force employed by the industrial and education sectors. We introduce

$$k_i(t) \equiv \frac{K_i(t)}{N_i(t)}, \, k_e(t) \equiv \frac{K_e(t)}{N_e(t)}.$$

As full employment of labor and capital is assumed, we have

$$N_i(t) + N_e(t) = N(t). \tag{3}$$

2.1 The industrial sector

We assume that production is to combine 'qualified labor force' $N_i(t)$ and physical capital, $K_i(t)$. We use the following production function $F_i(t)$ to describe a relationship between inputs and output

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad A_i, \alpha_i, \beta_i > 0, \, \alpha_i + \beta_i = 1.$$

Markets are competitive; thus labor and capital earn their marginal products. The rate of interest and wage rate are determined by markets. Hence, for any individual firm $r(t)$ and $w(t)$ are given at each point of time. The industrial sector chooses the two variables $K_i(t)$ and $N_i(t)$ to maximize its profit. The marginal conditions are given by

$$r(t) + \delta_k = \alpha_i A_i k_i^{-\beta_i}(t), \, w(t) = \beta_i A_i k_i^{\alpha_i}(t), \tag{4}$$

where δ_k is the fixed depreciation rate of physical capital.

2.2 Education sector

We assume that the education sector is characterized of perfect competition. Students pay the education fee $p_e(t)$ per unit of time. The education sector pays teachers and capital with the market rates. The total education service is measured by the total education time received by the population. We specify the production function of the education sector as follows

$$F_e(t) = A_e K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \quad A_e, \alpha_e, \beta_e > 0, \, \alpha_e + \beta_e = 1. \tag{5}$$

The marginal conditions for the education sector are

$$r(t) + \delta_k = \alpha_e A_e p_e(t) k_e^{-\beta_e}(t), \, w(t) = \beta_e A_e p_e(t) k_e^{\alpha_e}(t). \tag{6}$$

We see that the demand for labor force from the education sector increases in the price and level of human capital and decreases in the wage rate.

2.3 Money supply and distribution

With regard to outside money holdings as net wealth, Tobin (1965, p. 676) points out:

> "The community's wealth ... has two components: the real goods accumulated through past real investment and fiduciary or paper 'goods' manufactured by the government from thin air. Of course the non-human wealth of such a nation 'really' consists only of its tangible capital. But, as viewed by the inhabitants of the nation individually, wealth exceeds the tangible capital stock by the size of what we might term the fiduciary issue. This is an illusion, but only one of the many fallacies of composition which are basic to any economy or any society. The illusion can be maintained unimpaired so long as the society does not actually try to convert all of its paper wealth into goods."

To describe dynamics of money, we first assume that money is introduced by assuming that a central bank distributes at no cost to the population a per capita amount of fiat money $M(t) > 0$. The scheme according to which the money stock evolves over time is deterministic and known to all agents. With μ being the constant net growth rate of the money stock, $M(t)$ evolves over time according to

$$\dot{M}(t) = \mu M(t), \mu > 0.$$

The government expenditure in real terms per capita, $\tau(t)$, is given by

$$\tau(t) = \frac{\dot{M}(t)}{P(t)} = \frac{\mu M(t)}{P(t)} = \mu m(t), \tag{7}$$

where $P(t)$ is the price of money. The representative household receives $\mu m(t)$ units of paper money from the government through a "helicopter drop", also considered to be independent of the households' money holdings. From $M(t) = m(t)P(t)$, we have

$$\pi(t) = \frac{\dot{P}(t)}{P(t)} = \mu - \frac{\dot{m}(t)}{m(t)}, \tag{8}$$

where $\pi(t)$is the inflation rate.

2.4 Consumer behaviors and wealth dynamics

Different from the optimal growth theory in which utility defined over future consumption streams is used, we do not explicitly specify how consumers depreciate future utility resulted from consuming goods and services.[5] We assume that we can find preference structure of consumers over consumption and saving at the current state. In this study, we follow Zhang (2005, 2009) in modeling choice of education time and money to be held. The preference over current and future consumption is reflected in

[5] With regard to some limitations of the traditional Ramsey approach to household behavior, we refer to Attanasio and Weber (2010).

the consumer's preference for education, money, consumption and saving. Per household's current income from the interest payment, $r(t)\bar{k}(t)$, and the wage payments, $H^{\omega}(t)T(t)w(t)$, is given by

$$y(t) = r(t)\bar{k}(t) + T(t)\overline{w}(t) - \pi(t)m(t) + \tau(t),$$

where $\overline{w}(t) \equiv H^{\omega}(t)w(t)$ is the wage rate of qualified labor. The wage income is given by $\overline{W}(t) \equiv T(t)\overline{w}(t)$. The total value of wealth that a household can sell to purchase goods and to save is equal to $a(t)$, where $a(t) \equiv \bar{k}(t) + m(t)$. Here, we do not allow borrowing for current consumption. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. This is evidently a strict consumption as it may take time to draw savings from bank or to sell one's properties. The disposable income of a household is defined as the sum of the current income and the wealth available for purchasing consumption goods and saving, $\hat{y}(t) = y(t) + a(t)$. That is

$$\hat{y}(t) = a(t) + r(t)\bar{k}(t) + T(t)\overline{w}(t) - \pi(t)m(t) + \tau(t). \tag{9}$$

The disposable income is used for saving, home production, consumption, and education. At time t the consumer has the total amount of income equaling $\hat{y}(t)$ to distribute among saving, holding money, consuming, and receiving education.

Denote $T_e(t)$ the time spent on education. Let the (fixed) total available time be denoted by T_0. The time constraint is expressed by

$$T(t) + T_e(t) = T_0. \tag{10}$$

Insert (10) in (9)

$$\hat{y}(t) = \bar{y}(t) - \pi(t)m(t) - T_e(t)\overline{w}(t), \tag{11}$$

where

$$\bar{y}(t) \equiv a(t) + r(t)\bar{k}(t) + T_0\overline{w}(t) + \tau(t). \tag{12}$$

We interpret the variable $\bar{y}(t)$ as the potential disposable income as it is the disposable income $\hat{y}(t)$ when all the available time is spent on work (i.e., $T_j = T_0$).

The disposable income is spent on holding money $m(t)$, consuming the good $c(t)$, receiving education $T_e(t)$, and saving $s(t)$. We have

$$(1 + r(t))m(t) + c(t) + s(t) + p_e(t)T_e(t) = \hat{y}(t). \tag{13}$$

Insert (11) in (13)

$$\bar{\pi}(t)m(t) + c(t) + s(t) + \bar{p}(t)T_e(t) = \bar{y}(t), \tag{14}$$

where

$$\bar{\pi}(t) \equiv \pi(t) + r(t), \bar{p}(t) \equiv p_e(t) + \overline{w}(t).$$

Here, $\bar{\pi}(t)$ and $\bar{p}(t)$ are respectively the opportunity cost of holding money and the opportunity cost of receiving education. The consumer problem is to choose current money, education, consumption, and savings so that the utility is maximized.

The household's utility function is dependent on the money, education, consumption level of the goods, and saving in the following way

$$U(t) = m^{\varepsilon_0}(t)\, c^{\xi_0}(t)\, s^{\lambda_0}(t)\, T_e^{\eta_0}(t), \quad \varepsilon_0, \xi_0, \lambda_0, \eta_0 > 0,$$

where ε_0 is called propensity to hold money, ξ_0 the propensity to consume, λ_0 the propensity to own wealth, and η_0 the propensity to receive education. This utility function is applied to different economic problems. A detailed explanation of the approach and its applications to different problems of economic dynamics is provided in Zhang (2005, 2009). Here, we consider that education has two kinds of returns. As education raises labor productivity, its effect is reflected in higher wages. This return is reflected in the human capital accumulation equation and marginal condition of the two sectors. As Lazear (1977, p. 570) describes: "Education is simply a normal consumption good and that, like all other normal goods, an increase in wealth will produce an increase in the amount of schooling purchased. Increased incomes are associated with higher schooling attainment as the simple result of an income effect." Education also brings about direct pleasure, more knowledgeable, higher social status and so on (see, for instance, Heckman 1976; Lazear 1977; and Malchow-Møller et al. 2011). The relative importance of these returns may vary across different types of education with different individuals. The utility function takes account of the direct influence of education on utility. Maximizing $U(t)$ subject to (14) yields

$$\overline{\pi}(t)\, m(t) = \varepsilon \bar{y}(t),\, c(t) = \xi \bar{y}(t),\, s(t) = \lambda \bar{y}(t),\, \overline{p}(t)\, T_e(t) = \eta \bar{y}(t), \tag{15}$$

where

$$\varepsilon \equiv \rho \varepsilon_0,\, \xi \equiv \rho \xi_0,\, \lambda \equiv \rho \lambda_0,\, \eta \equiv \rho \eta_0,\, \rho \equiv \frac{1}{\varepsilon_0 + \xi_0 + \lambda_0 + \eta_0}.$$

The real wealth changes as follows

$$\dot{a}(t) = s(t) - a(t). \tag{16}$$

This equation means that the change in wealth is equal to the savings minus the dissavings.

2.5 Dynamics of human capital

Different forms of learning have different effects of human capital accumulation (Aakvik et al. 2010). For instance, social and economic conditions of the family and school quality are crucial to educational achievement (Kane 1994; Maurin 2002; Dearden et al. 2002). In the Uzawa-Lucas model and many of their extensions and generalizations, human capital is formed due to formal schooling. Ignoring these non-school factors may make us misunderstand the role of formal education in economic development. Except learning through formal education, this study takes account of learning by producing and learning by leisure. Arrow (1962) formally took account of learning by doing in studying economic growth; Uzawa (1965) first modeled trade-offs between investment in education and capital accumulation; and Zhang (2007) first considered the impact of consumption on human capital accumulation (via the so-called creative leisure) in the neoclassical growth theory with endogenous human and physical accumulation. As in Zhang (2007), the human capital dynamics is

$$\dot{H}(t) = \frac{\upsilon_e F_e^{a_e}(t)\left(H^{\omega}(t)\,T_e(t)\overline{N}\right)^{b_e}}{H^{\pi_e}(t)\overline{N}} + \frac{\upsilon_i F_i^{a_i}(t)}{H^{\pi_i}(t)\overline{N}} + \frac{\upsilon_c c^{a_c}(t)}{H^{\pi_c}(t)} - \delta_h H(t), \tag{17}$$

where δ_h is the depreciation rate of human capital, υ_e, υ_i, υ_c, b_e, a_i, and a_c are non-negative parameters. The signs of the parameters π_e, π_i, and π_c are not specified as they may be either negative or positive.

The above equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation. The term, $\upsilon_e F_e^{a_e}\left(H^m T_e \overline{N}\right)^{b_e} / H^{\pi_e}\overline{N}$, describes the contribution to human capital improvement through education. Human capital increases with a rise in the level of education F_e, and in the (qualified) total study time, $H^m T_e \overline{N}$. The population in the denominator measures the contribution in terms of per capita. The term H^{π_e} indicates that as the level of human capital of the population increases, it may be more difficult (in the case of $\pi_e > 0$) or easier, for instance, due to learning externalities as in Choi (2011) (in the case of $\pi_e < 0$) to accumulate more human capital via formal education.[6]

We take account of learning by doing effects in human capital accumulation by the term $\upsilon_i F_i^{a_i} / H^{\pi_i}$. This term implies that contribution of the production sector to human capital improvement is positively related to its production scale F_i and is dependent on the level of human capital. The term H^{π_i} takes account of returns to scale effects in human capital accumulation. The case of $\pi_i > 0$ ($\pi_i < 0$) implies that as human capital is increased it is more difficult (easier) to further improve the level of human capital.

We take account of learning by consuming by the term $\upsilon_c c^{a_c} / H^{\pi_c}$. If one consumes more, one's human capital tends to be increased. If one, for instance, uses advanced computers and other home equipment, owns a great variety of books, travels frequently at home and abroad, one tends to accumulate human capital effectively. Nevertheless, there may be decreasing return to scale in human capital accumulation through leisure and consumption. For instance, a man born into rich may learn little by consuming more as the knowledge/skills which may be accumulated through consuming is already mastered through earlier consumption.

It should be noted that in the theoretical literature on education and economic growth, possible sources of learning such as learning by doing and learning by consuming are not explicitly taken into account. In most of the growth models with endogenous physical and human capital accumulation, only formal education (including job training in some studies) is considered. It is commonly assumed that human capital evolves according to the following equation (see Barro and Sala-i-Martin 1995)

$$\dot{H}(t) = H^{\eta}(t)\,G(T_e(t)),$$

where the function G is increasing as the effort rises with $G(0) = 0$. There is neither physical input nor teachers' efforts in the model. When $\eta < 1$, there is diminishing

[6] See also, Rauch (1993) and Liu (2007), for the literature on human capital externalities. With regard to economies of scale and scope in education, see Cohn and Cooper (2004).

return to the human capital accumulation. This formation is proposed by Lucas (1988). As $\dot{H}/H < H^{\eta-1}G(1)$, the growth rate of human capital must eventually tend to zero no matter how much effort is devoted to accumulating human capital.

Uzawa's model is a special case of the Lucas model with $\gamma = 0$, $U(c) = c$, and the assumption that the right-hand side of the above equation is linear in the effort. It seems reasonable to consider diminishing returns in human capital accumulation: people accumulate it rapidly early in life, then less rapidly, then not at all—as though each additional percentage increment were harder to gain than the preceding one.

Solow (2000) uses the following form: $\dot{H}(t) = H(t)\,\kappa T_e(t)$. This is a special case of the above equation. The new formation implies that if no effort is devoted to human capital accumulation, then $\dot{H}(0) = 0$ (human capital does not vary as time passes; this results from depreciation of human capital being ignored); if all effort is devoted to human capital accumulation, then $g_H(t) = \kappa$ (human capital grows at its maximum rate; this results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$. To achieve a given percentage increase in $H(t)$ requires the same effort.

As remarked by Solow, the above formulation is very far from a plausible relationship. If we consider the above equation as a production for new human capital (i.e., $\dot{H}(t)$), and if the inputs are already accumulated human capital and study time, then this production function is homogenous of degree two. It has strong increasing returns to scale and constant returns to $H(t)$ itself. It can be seen that our approach is more general to the traditional formation with regard to education. Moreover, we treat teaching also as a significant factor in human capital accumulation. Efforts in teaching are neglected in Uzawa-Lucas model. In a recent study by Choi (2011) (where capital input is neglected), human capital accumulation equation is

$$\dot{H}(t) = B(t)\left[u(t)H(t)\right]^{\varphi}\overline{H}^{\theta}(t) - \delta_H H(t),$$

where $B(t)$ is an exogenous time-dependent variable (called productivity of human capital production) and $u(t)$ is the fraction of human capital devoted to human capital accumulation. The new variable, $\overline{H}(t)$, is the average human capital stock in the economy. The term, $\overline{H}^{\theta}(t)$, measures learning externalities. As for a homogenous population, $\overline{H}(t)$ is $H(t)$. We see that Choi's learning equation is a special case of (17).

2.6 Demand of and supply for education

For the education sector, the demand and supply balances at any point of time

$$T_e\overline{N} = F_e(t). \tag{18}$$

We have thus built the dynamic model. We now examine dynamics of the model.

3. The motion and equilibrium of the economic system

We first show that the dynamics are determined by three differential equations. The following lemma is proved in the Appendix.

Lemma 1. *The motion of the economic system is described by the following three differential equations with $k_i(t)$, $m(t)$, and $H(t)$, as the variables*

$$\dot{m} = \Lambda_m(m, k_i, H),$$

$$\dot{k}_i = \Lambda_{k_i}(m, k_i, H),$$

$$\dot{H} = \Lambda_H(m, k_i, H), \tag{19}$$

where Λ_m, Λ_m, and Λ_{k_i} are functions $m(t)$, $k_i(t)$, and $H(t)$ defined in the Appendix. All the other variables are determined as functions of k_i, m, and H at any point of time by the following procedure: r and w by (4) $\rightarrow \bar{k}$ *by* (A13) $\rightarrow k_e$ *by* (A1) $\rightarrow \bar{y}$ *by* (A3) $\rightarrow N$ *by* (A7) $\rightarrow N_i$ *by* (A8) $\rightarrow N_e$ *by* (3) $\rightarrow K = \bar{k}\overline{N} \rightarrow K_i = k_i N_i \rightarrow K_e = k_e N_e \rightarrow p_e$ *by* (A2) $\rightarrow \pi$ *by* (15) $\rightarrow$ c, s, *and* T_e *by* (15) $\rightarrow T$ *by* (10) $\rightarrow F_i = A_i K_i^{\alpha_i} N_i^{\beta_i} \rightarrow F_e$ *by* (5).

This lemma is important as it tells us how to follow the motion of the economic system, given proper initial conditions. With computer it is straightforward to plot the motion of the dynamic economic system. As the expressions are too tedious, we cannot easily interpret the analytical results. For illustration, we simulate the model, specifying the parameter values of the two sectors as follows

$$\overline{N} = 5,\ T_0 = 1,\ \alpha_i = 0.35, A_i = 1,\ \alpha_e = 0.45, A_e = 1,\ \mu = 0.02,\ \delta_k = 0.05.$$

Although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. For instance, some empirical studies on the US economy demonstrate that the value of the parameter, α, in the Cobb-Douglas production is approximately equal to 0.3. (see, for instance, Miles and Scott 2005 and Abel et al. 2007). With regard to the technological parameters, what are important in our study are their relative values. This is similarly true for the specified preference parameters. The household's preference and utilization efficiency of human capital

$$\lambda_0 = 1,\ \xi_0 = 0.1,\ \eta_0 = 0.02,\ \varepsilon_0 = 0.02,\ \omega = 0.7.$$

The efficiency of human capital utilization is $\omega = 0.7$. That is, the efficiency elasticity of human capital is $\omega = 0.7$. The propensity to receive education is 0.02. The parameters of the human capital accumulation equation are specified as

$$\begin{aligned} &v_e = 1, v_i = 0.8, v_c = 0.5, a_e = 0.3, a_i = 0.3, a_c = 0.3, b_e = 0.5, \pi_e = -0.1, \\ &\pi_i = 0.7, \pi_c = 0.1, \delta_h = 0.08. \end{aligned} \tag{20}$$

According to Stokey and Rebelo (1995), it is reasonable to consider the depreciation rate of human capital a range between 0.03 and 0.08 for the US economy. We demonstrate that with the above specified parameters, the system has a unique equilibrium point. The equilibrium values of the variables of the two sectors are

$$Y = 69.08, g = 13.82, H = 18.06, N = 31.62, N_i = 31.37, N_e = 0.25,$$

$$K = 297.83, K_i = 291.29, K_e = 3.55, F_i = 68.43, F_e = 0.83, C = 47.79,$$

$$k_i = 9.29,\, k_e = 14.11,\, f_i = 2.18,\, f_e = 3.29,$$

where $Y \equiv F_i + p_e F_e$ and $g \equiv Y/\overline{N}$ are respectively the national output and per capita GDP $Y \equiv F_i + p_e F_e$. We see that most of the labor force and capital stocks are employed by the industrial sector. The industrial sector's capital intensity is lower than the education sector's capital intensity. The prices are

$$r = 0.032,\, p_e = 0.78,\, \overline{w} = 10.75,\, \overline{W} = 8.97,\, \pi = 0.02.$$

The values of the variables describing the household are

$$\bar{k} = 58.97,\, T = 0.83,\, T_e = 0.17,\, c = 9.56,\, m = 36.61.$$

The physical wealth is higher with the money holdings. The household spends 17 per cent of the available time on education. The ratio of the money holding to the physical wealth is about 60 per cent. The three eigenvalues are -0.13, 0.09, -0.04. The equilibrium point is a saddle point. We specify the following initial conditions

$$k_i(0) = 11,\, m(0) = 32,\, H(0) = 20.$$

The changes of the variables over time are plotted in Figure 1. As the model has a saddle point, the system normally does not converge to the equilibrium point. The economic system either grows infinite or collapses if it does not start at a proper initial condition. Hence, we stimulate the model with a short period of time. The real rate of interest falls and inflation rate rises over time. The national output level and per capita GDP rise, even though human capital falls partly as a consequence of fall in household's study time. This occurs because the worker is devoted more to work. The net result of the fall in human capital and the rise in work time leads to the rise in the GDP. The physical wealth is increased even when the rate of interest falls. Money held is reduced partly as a consequence of sped-up inflation. As the total qualified labor input falls, the labor inputs of the two sectors are reduced.

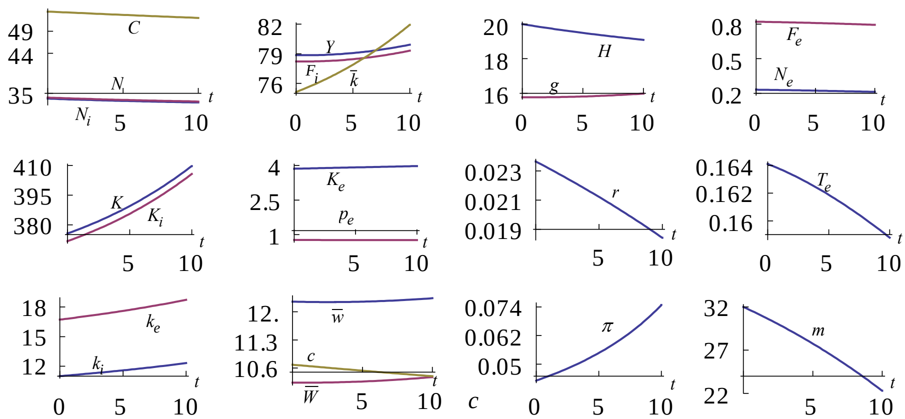

Figure 1. The motion of the system with money and division of labor

4. Comparative static analysis

The previous section identifies the unique equilibrium of the dynamic economy and demonstrates that the economic system has a saddle point. This section examines impact of changes in some parameters on the long term equilibrium point. First, we examine the case that all the parameters, except the inflation policy μ, are the same as in (20). We increase μ from 0.02 to 0.03. The simulation results are listed in (21):

$$\overline{\Delta}Y = \overline{\Delta}g = 4.11, \overline{\Delta}H = 1.29, \overline{\Delta}N = 0.71, \overline{\Delta}N_i = 0.74, \overline{\Delta}N_e = -326, \overline{\Delta}K = 10.74,$$
$$\overline{\Delta}K_i = 10.79, \overline{\Delta}K_e = 6.39, \overline{\Delta}F_i = 4.15, \overline{\Delta}F_e = 0.97, \overline{\Delta}C = 4.97, \overline{\Delta}k_i = \overline{\Delta}k_e = 9.98,$$
$$\overline{\Delta}f_i = 3.38, \overline{\Delta}f_e = 4.37, \overline{\Delta}r = -15.30, \overline{\Delta}p_e = -0.95, \overline{\Delta}\overline{w} = 4.32, \overline{\Delta}\overline{W} = 4.12,$$
$$\overline{\Delta}\pi = 50, \overline{\Delta}\overline{k} = 10.74, \overline{\Delta}T = -0.19, \overline{\Delta}T_e = 0.97, \overline{\Delta}c = 4.97, \overline{\Delta}m = -4.32 \qquad (21)$$

Here, symbol $\overline{\Delta}$ stands for the change rate due to the parameter change. Our model predicts that money is not neutral as a change in the inflation policy will change all the real variables as well as monetary variables. It should be noted that according to in the literature of monetary growth there is no uniform answer to the question of effects of monetary policy.

As found by Kam and Mohsin (2006, p. 52), the results depend on "the methods for modeling money in the general equilibrium framework; money in the utility function (MIU) and cash-in-advance constraints (CIA). With MIU, time preference wealth effects link the monetary and real sectors by endogenizing real interest rate. Monetary growth raises steady state capital and consumption by the Tobin effect. However, if money is introduced through CIA constraints, inflation policies are sensitive to the structure of the constraint itself. If the constraint applies to consumption and capital purchases, monetary growth lowers the steady state demand for both commodities and reverses the Tobin effect."

In our approach, we see that a rise in the inflation policy raises the national output and per capita GDP. This conclusion is similar to those in Kam and Mohsin (2006) with the MIU approach. It should be noted that there is no endogenous human capital in Kam and Mohsin (2006). The inflation policy in our approach has also the real effects on the economic structure and human capital accumulation. We see that the study time is increased in association of falling price of education. Human capital is also increased since the education time is increased. The education and industrial sectors' levels of output, and consumption level are all increased. The wage rate of the qualified labor and the wage income are increased. The money holding is reduced partly as a consequence of decreases in the rate of interest and the inflation rate.

We now examine the impact of a rise in the propensity to hold money from 0.02 to 0.025. The effects are listed in (22):

$$\overline{\Delta}Y = \overline{\Delta}g = -5.31, \overline{\Delta}H = -1.98, \overline{\Delta}N = -1.05, \overline{\Delta}N_i = -1.09, \overline{\Delta}N_e = 3.98,$$
$$\overline{\Delta}K = -12.73, \overline{\Delta}K_i = -12.78, \overline{\Delta}K_e = -8.31, \overline{\Delta}F_i = 5.35, \overline{\Delta}F_e = -1.74, \overline{\Delta}C = -6.82,$$
$$\overline{\Delta}k_i = \overline{\Delta}k_e = -11.82, \overline{\Delta}f_i = -4.31, \overline{\Delta}f_e = -4.51,$$

$$\bar{\Delta} r = 21.75,\ \bar{\Delta} p_e = 1.27,\ \bar{\Delta} \overline{w} = -5.64,\ \bar{\Delta} \overline{W} = -5.31,\ \bar{\Delta} \pi = 0,$$
$$\bar{\Delta} \overline{k} = -12.73,\ \bar{\Delta} T = 0.35,\ \bar{\Delta} T_e = -1.74,\ \bar{\Delta} c = -6.82,\ \bar{\Delta} m = 2.69 \tag{22}$$

As the household prefers more to hold money, the money is increased. The national output and per capita GDP are reduced. As the preference for holding money is strengthened, the propensity to receive education is relatively weakened. Partly as a consequence of the reduced propensity to receive education, the study time is reduced. The net result of an increase in work time and a fall in human capital leads to a fall in the qualified labor supply. The rate of interest rises in association with a rise in the total capital. The two sectors' capital inputs are reduced.

We now examine effects of changes in the household's preference for education. According to Krueger and Lindahl (2001), each additional year of schooling appears to increase earnings by about 10 per cent in U.S. They also show that the rate of return to education varies across countries. We increase the household's propensity to receive education η_0 from 0.02 to 0.03. The effects are listed in (23):

$$\bar{\Delta} Y = \bar{\Delta} g = 2.40,\ \bar{\Delta} H = 16.18,\ \bar{\Delta} N = 2.49,\ \bar{\Delta} N_i = 2.20,\ \bar{\Delta} N_e = 39.09,\ \bar{\Delta} K = 2.24,$$
$$\bar{\Delta} K_i = 1.79,\ \bar{\Delta} K_e = 38.54,\ \bar{\Delta} F_i = 2.06,\ \bar{\Delta} F_e = 38.85,\ \bar{\Delta} C = 1.98,\ \bar{\Delta} k_i = \bar{\Delta} k_e = -0.40,$$
$$\bar{\Delta} k_i = \bar{\Delta} k_e = -0.40,\ \bar{\Delta} f_i = -0.14,\ \bar{\Delta} f_e = -0.18,\ \bar{\Delta} r = 0.66,\ \bar{\Delta} p_e = 0.04,\ \bar{\Delta} \overline{w} = 10.91,$$
$$\bar{\Delta} \overline{W} = 2.35,\ \bar{\Delta} \pi = 0,\ \bar{\Delta} \overline{k} = 2.24,\ \bar{\Delta} T = -7.72,\ \bar{\Delta} T_e = 38.85,\ \bar{\Delta} c = 1.98,\ \bar{\Delta} m = 1.57 \tag{23}$$

As the household prefers to spend more time and resource on education, the study time is increased and human capital is improved. If the household in an economy likes to study, the national output, per capita GDP, the inputs of capital and work forces in the two sectors, and the output levels of the two sectors, the wage rate, and the wage income are all increased. Hence, if a population loves to learn and applies knowledge effectively, the national economy tends be highly developed.

This paper treats education as service. We analyzed the effects in the demand side on the real and monetary variables. We now examine effects of change in the supply size. We increase the education sector's total productivity A_e from 1 to 1.1. The effects are listed in (24):

$$\bar{\Delta} Y = \bar{\Delta} g = 0.09,\ \bar{\Delta} H = 0.27,\ \bar{\Delta} N = 0.07,\ \bar{\Delta} N_i = 0.14,\ \bar{\Delta} N_e = -8.58,\ \bar{\Delta} K = 0.13,$$
$$\bar{\Delta} K_i = 0.24,\ \bar{\Delta} K_e = -8.49,\ \bar{\Delta} F_i = 0.18,\ \bar{\Delta} F_e = 0.60,\ \bar{\Delta} C = 0.19,\ \bar{\Delta} k_i = \bar{\Delta} k_e = 0.10,$$
$$\bar{\Delta} f_i = 0.03,\ \bar{\Delta} f_e = 10.05,\ \bar{\Delta} r = -0.16,\ \bar{\Delta} p_e = -9.10,\ \bar{\Delta} \overline{w} = 0.23,\ \bar{\Delta} \overline{W} = 0.11,$$
$$\bar{\Delta} \pi = 0,\ \bar{\Delta} \overline{k} = 0.13,\ \bar{\Delta} T = -0.12,\ \bar{\Delta} T_e = 0.60,\ \bar{\Delta} c = 0.19,\ \bar{\Delta} m = 0.29 \tag{24}$$

As the supply becomes more effective, the price of education falls. As education becomes economically more available, the household spends more time on education. The increase in education time increases human capital and wage income. The net result in the fall in the work time and the rise in human capital results in the rise on the total qualified labor supply. The national output, per capita GDP, the inputs of capital and work forces in the industrial sector, and the output levels of the two sectors, the wage rate, and the wage income are all increased.

5. Conclusions

This study built a monetary growth model with inflation policy and education. The model is a synthesis of the Uzawa-Lucas two-sector growth model and traditional monetary model with the money-in-utility (MIU) approach. We show how money, physical capital and human capital interact over time under exogenous inflation policy in a free market economy.

The dynamics of the economy is described by three differential equations. We show that the monetary economic system has a saddle equilibrium point. We simulate motion of the economic system and carry out comparative statics analysis with regards to the monetary policy, propensities to save wealth and to receive education. Our comparative statics analysis provides some insights into effects of inflation policy, propensity to receive education and propensity to hold money.

For instance, money is not neutral in our model as a change in the inflation policy changes all the real and monetary variables. A rise in the inflation policy raises the national output and per capita GDP. This conclusion is similar to those in Kam and Mohsin (2006) with the MIU approach. The inflation policy has also the real effects on the economic structure and human capital accumulation. We see that the study time is increased in association of falling price of education. Human capital is increased since the education time is increased. The education and industrial sectors' levels of output, and consumption level are all increased. The wage rate of the qualified labor and the wage income are increased. The money holding is reduced partly as a consequence of decreases in the rate of interest and the inflation rate.

We may generalize and extend our model in different ways. It is well known that one-sector growth model has been generalized and extended in many directions. It is not difficult to generalize our model along these lines. It is straightforward to develop the model in discrete time. We may analyze behavior of the model with other forms of production or utility functions.

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Appendix

Proof of Lemma 1.

From (4) and (6), we obtain

$$k_e = \alpha k_i, \tag{A1}$$

where $\alpha \equiv \alpha_e \beta_i / (\alpha_i \beta_e)$ ($\neq 1$ assumed) and the time index is suppressed wherever no confusion. From (A1), (4) and (6), we obtain

$$p_e = \frac{\alpha^{\beta_e} \alpha_i A_i}{\alpha_e A_e} k_i^{\beta}, \tag{A2}$$

where $\beta \equiv \beta_e - \beta_i$. We note that r and w are uniquely determined as functions of k_i by (4). From the definition of $\bar{y}$ we have

$$\bar{y} = R\bar{k} + \overline{W}, \tag{A3}$$

where we use $R \equiv 1 + r$, $\overline{W} = T_0 \overline{w} + (1+\mu) m$. From (18) and $w = \beta_e p_e F_e / N_e$ we have

$$T_e = \frac{w N_e}{\beta_e p_e \overline{N}}. \tag{A4}$$

From (A4) and $T_e = \eta \bar{y} / \bar{p}$ in (15), we have

$$N_e = \frac{\eta \overline{N} \beta_e p_e \bar{y}}{w \bar{p}}. \tag{A5}$$

From (12) and (15), we have

$$T = T_0 - \frac{\eta \bar{y}}{\bar{p}}. \tag{A6}$$

From (1) and (A6), we have

$$N = \left(T_0 - \frac{\eta \bar{y}}{\bar{p}} \right) H^{\omega} \overline{N}.) \tag{A7}$$

From (A7) and (3), we solve

$$N_i = \overline{N} T_0 H^{\omega} - \overline{\phi}\, \bar{y}, \tag{A8}$$

where

$$\overline{\phi}\,(k_i, H) \equiv \left(H^{\omega} + \frac{\beta_e p_e}{w} \right) \frac{\eta \overline{N}}{\bar{p}}.$$

From (A3), $\pi = \mu - \dot{m}/m$ and $(r + \pi) m = \varepsilon \bar{y}$, we obtain

$$\dot{m} = (r + \mu - (1+\mu)\varepsilon) m - \varepsilon R \bar{k} - \varepsilon T_0 \overline{w}. \tag{A9}$$

From (16) and the definition of $s(t)$, we have

$$\dot{\bar{k}} = (\lambda R - 1)\bar{k} + \lambda \overline{W} - m - \dot{m}. \tag{A10}$$

From (2) and $K = \bar{k}\overline{N}$, we have

$$\bar{k}\overline{N} = k_i N_i + k_e N_e. \tag{A11}$$

Insert (A5) and (A8) in (A11)

$$\bar{k}\overline{N} = \overline{N} T_0 k_i H^{\omega} + \tilde{\phi}\bar{y}, \tag{A12}$$

where

$$\tilde{\phi}(k_i, H) = \frac{\eta \overline{N} \beta_e k_e p_e}{w\bar{p}} - k_i \bar{\phi}.$$

From (A3) and (A12), we have

$$\bar{k} = \Phi(k_i, m, H) \equiv \frac{\overline{N} T_0 k_i H^{\omega} + \tilde{\phi}\overline{W}}{\overline{N} - \tilde{\phi} R}. \tag{A13}$$

Hence, we can express explicitly $\bar{k}$ as functions of k_i, m, H. We can now express all the variables as functions of k_i, m, and H at any point of time by the following procedure: r and w by (4) $\rightarrow \bar{k}$ by (A13) $\rightarrow k_e$ by (A1) $\rightarrow \bar{y}$ by (A3) $\rightarrow N$ by (A7) $\rightarrow N_i$ by (A8) $\rightarrow N_e$ by (3) $\rightarrow K = \bar{k}\overline{N} \rightarrow K_i = k_i N_i \rightarrow K_e = k_e N_e \rightarrow p_e$ by (A2) $\rightarrow \pi$ by (15) $\rightarrow c$, s, and T_e by (15) $\rightarrow T$ by (10) $\rightarrow F_i = A_i K_i^{\alpha_i} N_i^{\beta_i} \rightarrow F_e$ by (5). From this procedure and (17), we have

$$\dot{H} = \Lambda_H(k_i, m, H). \tag{A14}$$

Here, we don't provide explicit expressions of the functions as they are tedious. Insert (A13) in (A10)

$$\dot{m} = \Lambda_m(k_i, m, H) \equiv (r + \mu - (1+\mu)\varepsilon) m - \varepsilon R \bar{k} - \varepsilon T_0 \bar{w}. \tag{A15}$$

Take derivatives of (A13) with respect to time

$$\dot{\bar{k}} = \frac{\partial \Phi}{\partial k_i}\dot{k}_i + \frac{\partial \Phi}{\partial m}\Lambda_m + \frac{\partial \Phi}{\partial H}\Lambda_H, \tag{A16}$$

where we also use (A15) and (A14). From (A1), (A13), and (A15), we have

$$\dot{\bar{k}} = (\lambda R - 1)\Phi + \lambda \overline{W} - m - \Lambda_m. \tag{A17}$$

Equaling the right-hand sizes of (A17) and (A16), we get

$$\dot{k}_i = \Lambda_{k_i}(k_i, m, H) \equiv \left[(\lambda R - 1)\Phi + \lambda \overline{W} - m - \Lambda_m - \frac{\partial \Phi}{\partial m}\Lambda_m - \frac{\partial \Phi}{\partial H}\Lambda_H\right]\left(\frac{\partial \Phi}{\partial k_i}\right)^{-1}. \tag{A18}$$

In summary, we proved Lemma 1. □