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Endogenous Heterogeneity, the Propagation of Information and Macroeconomic Complexity

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Abstract The economy is an adaptive and evolving complex system. The recognition of this fact has produced, over the last few years, a gradual but firm shift on the direction taken by mainstream economic thought. The representative agent paradigm is giving place to settings of interacting heterogeneous agents, capable of generating dynamic outcomes that are unique for each assumed pattern of interaction. The complexity era, as it is called, requires original insights and innovative approaches, in order to reinterpret the economic reality. This paper intends to contribute to the complexity literature in economics by exploring a popular macroeconomic model (the sticky-information model) from a new angle. Information acquisition by price-setting firms will be endogenously determined in two steps: (i) the competition between almost identical media leads to an irregular pattern of information creation; (ii) once created, information spreads throughout the interested audience following a diffusion process. The consequence is an erratic aggregate pattern of information acquisition that has consequences at a macro level, namely concerning the shape of the Phillips curve, which will change every period. Heterogeneity, self-organization, evolution and out-of-equilibrium dynamics are features that allow to classify the suggested framework as a setting of macroeconomic complexity.

Keywords Complexity, information production and diffusion, heterogeneity, sticky-information, nonlinear dynamics
JEL classification E30, E52, D83, D84

1. Introduction

For many years, economic analysis has been confined to the use of relatively simple logical models that have provided relevant insights and allowed this science to progress, but that basically failed in putting into perspective the economy as being an evolving and complex system. The mechanicist view or neoclassical perspective, according to which a representative agent is able to provide all the necessary behavioral structure to understand how economic phenomena evolve, is being increasingly challenged in the profession. Authors like Rosser (1999), Velupillai (2005), Colander (2008), Fontana (2010), Holt et al. (2010) and Puu (2010), just to cite a few, call for the necessity of a paradigm change, with the determinism of the traditional approach giving place to a field of new ideas, techniques and approaches that share a same notion of interaction within a system populated by agents with different beliefs, endowments

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and expectations. This system is necessarily an environment of complexity.

In Arthur et al. (1997), Martin and Sunley (2007) and Fontana (2008), the main features of a complex economic system are listed and characterized. We may synthesize these into a brief set of ideas:

(i) *Heterogeneous agents*. Agents are endowed with distinct goals, skills, expectations, preferences, wealth levels, ...

(ii) *Interaction*. Because agents are heterogeneous, understanding aggregate phenomena requires analyzing patterns of interaction. No average behavior can capture the richness of the multiple social and economic networks that are formed when people and organizations interact.

(iii) *Evolution*. Within a complex interaction system, agents adapt and learn, i.e., they evolve.

(iv) *Path-dependency*. Heterogeneity and interaction generate events that are unique. The state of a system at a moment t only exists because of what happened in the sequence of time periods that preceded t. History determines the current state and because history does not repeat itself one should not expect a same series of events to happen twice.

(v) *Self-organization*. The invisible hand continues to be a fundamental feature of the relation between agents. There is no need for an external entity to regulate the interaction among agents within a complex system.

(vi) *Out-of-equilibrium dynamics*. Interaction among heterogeneous agents typically leads to the observation of everlasting endogenous fluctuations. A complex system does not tend to rest, in the long-term, in a stable position. Instead, a bounded instability evolution should characterize the behavior of economic aggregates in such a setting.

The question we ask in this paper is how much must we depart from typical economic analysis in order to generate a setting with the above properties. Advocates of the complexity approach tend to reject conventional economic models, adopting radically different tools to understand economic phenomena. The use of complex adaptive systems (Markose 2005) or the use of tools from Econophysics (Mantegna and Stanley 2000; Rosser 2008) are two examples of such radical departures. We will argue that the mentioned set of properties may be highlighted within the boundaries of a much more conventional economic model; we will make use of the sticky-information monetary policy model, first presented in Mankiw and Reis (2002) and further analyzed in Mankiw and Reis (2003, 2006, 2007) and Reis (2009).

The mentioned model involves a particular form of agents' heterogeneity: firms update information on the state of the economy, in order to take price-setting decisions, at different dates and, as a result, expectations about current prices may have been formulated at different time moments in the past. However, this element of heterogeneity is hidden behind the assumption that each firm has a same probability of being one of the firms updating its information independently of the date of the previous change. This eliminates heterogeneity on the aggregate, making it feasible to characterize the macroeconomic system through a set of relations that are time invariant. The model is built in such a way that we do not know which firms update their information at time t, but we know for certain that a given share of firms will proceed with the process of collecting and treating information, in order to generate more precise forecasts on future events, at this precise time period. As a result, a single parameter is able to translate the degree of information stickiness that the economy is exposed to, and heterogeneous behavior fades out when taking a macro perspective.

Instead of establishing a time invariant rule for aggregate information updating, we will allow this to take a different shape at distinct time moments. This becomes possible when we start exploring the reasons why firms eventually access (or do not access) new information at each time moment. The framework will take into account the interaction between media companies; these companies act in the same way but they may have slightly different initial levels of informational resources. This minor difference triggers a process of cyclical information release. Because the flow of generated information is not linear, the share of agents accessing information at each time period will not be linear as well, and it will follow an irregular pattern. The endogenous volatility in information updating by price-setting firms has, as main consequence, the formation of different supply side relations for distinct time periods. We will be able to determine a differently shaped Phillips curve relation for each one of the considered calendar dates; information stickiness will no longer be translated into a single constant parameter value.

The suggested scenario will comprehend much of the properties one has advanced to characterize a complex system: heterogeneity is the element that triggers the analysis, the interaction between media units, on one hand, and between productive firms, on the other hand, is capital for the evolution of the macroeconomic system. Agents adapt to the behavior of others and trajectories of macro variables become unique for the specific process of interaction that is established leading to a self-organized economy that does not converge to a stable regular long-run state of equilibrium. Under these characteristics, one can associate the studied setup to a complex environment. One important argument is that a minimal heterogeneity assumption may be sufficient to trigger the endogenous fluctuations setting that is obtained.

The remainder of the paper is organized as follows. Section 2 describes how information is generated through the process of interaction between two competing media companies. In Section 3, it is explained how the created information reaches the interested audience (price-setting firms). Section 4 characterizes the macroeconomic environment, by aggregating the behavior of individual firms. In Section 5, we compute Phillips curves; these are different for each assumed time period, given the different shares of attentive firms we observe at each period. In Section 6, we introduce the demand into the analysis and evaluate the implications of a changing supply relation over the economy as a whole. Finally, Section 7 concludes.

2. Information: common knowledge, externalities and (almost) identical media

In this section, we build a framework for the analysis of the generation of new information. We assume two media units (two newspapers, two TV news channels, two web sites, ...) that compete and cooperate in order to create information that is useful for economic agents and, in particular, for price-setting firms. The proposed environment combines ideas from the knowledge creation setup in Fujita (2009) and from the competing technologies setting in Gomes (2008).

We start by defining ω_t^a and ω_t^b as indexes of quality of the information broadcasted, respectively, by media companies **A** and **B**. Our analysis is intertemporal and, thus, these indexes evolve in time possibly assuming different values at each different time period t. The information held by each unit possesses two components: differential information, that is known only from one specific media unit, ω_t^{ad} and ω_t^{bd}, and common information, which the two companies share, ω_t^c. The shared information allows the two entities to communicate and to create a synergy that leads to a refinement of the quality of the information generated by both units. Thus, the value of the information of the two media can be presented as

$$\omega_t^a = \omega_t^{ad} + \omega_t^c, \quad \omega_t^b = \omega_t^{bd} + \omega_t^c.$$

Media companies work partially in isolation and partially in cooperation with the other company. The processes through which they increase the value of the generated information differs from one case to the other. Consider first the isolation case. We assume identical media units and, therefore, the process of accumulation of increasing quality information is identical between them. This process is given by the following expressions:

$$\omega_{t+1}^{ad} = (1-\delta)\omega_t^{ad} + f(\omega_t^a)\xi^a(\omega_t^a, \omega_t^b) \tag{1}$$

$$\omega_{t+1}^{bd} = (1-\delta)\omega_t^{bd} + f(\omega_t^b)\xi^b(\omega_t^a, \omega_t^b) \tag{2}$$

The above expressions indicate that differentiated information in $t+1$ is a share of the period t information index ($\delta \in (0,1)$ is a rate of information obsolescence) plus a term indicating how new information is generated; f will be a continuous and differentiable function such that $f' > 0$ and $f'' < 0$; ξ^i, $i = a, b$ will, in turn, represent externalities that influence the process of creation of new information. In order to make the model tractable, we consider the following functional form for f: $f(\omega_t^i) = A(\omega_t^i)^\rho$, $i = a, b$, with $A > 0$ an efficiency value (it represents the contribution of other variables besides information in the generation of new information) and $\rho \in (0,1)$ an elasticity parameter. Concerning the externality function, the following will be considered

$$\xi^a(\omega_t^a, \omega_t^b) = 1 + \eta \arctan(\omega_t^b - \omega_t^a)$$

$$\xi^b(\omega_t^a, \omega_t^b) = 1 + \eta \arctan(\omega_t^a - \omega_t^b)$$

with $\eta > 0$. These externality functions indicate that if the value of the information of the other media unit is larger than the one respecting the considered company, this has a positive effect on the process of generation of new information (in this case, $\xi^i > 1$); if the opposite occurs, there is a pernicious influence of the other unit's information over the assumed process of information creation and, thus, the externality will be negative ($\xi^i < 1$). When both units generate information with identical quality, $\omega_t^a = \omega_t^b$, then no externality will be observable ($\xi^i = 1$). These functions are similar to the ones presented in Gomes (2008).

Note that both in the production function and in the externality function, it is the overall information held by the media that is relevant to develop additional differentiated information. If the externalities are too strong, this can generate a decrease in the value of information and the differential information values can even become negative. In this case, we might assert that the produced information is of a such poor quality that the interested audiences will be better off if they do not access that information.

Having characterized the process of creation of differentiated information, we now need to look at common information. Here, the basic principle is that collaboration between media units requires a reasonable balance between differentiated and common knowledge. Common knowledge is required for the two units to be able to communicate; differential information is needed in order to be advantageous for the media to meet and collaborate. As referred in Fujita (2009), too much common information implies a low level of heterogeneity in the collaboration, while excessively differentiated levels of information do not allow for a common ground to flourish. Hence, the production of new common information requires a balance between originality and shared knowledge. Based on the previous arguments, we will consider that the process of accumulation of common information will take the following shape:

$$\omega_{t+1}^c = (1-\delta)\omega_t^c + g(\omega_t^{ad}, \omega_t^{bd}, \omega_t^c) \tag{3}$$

According to (3), common information is subject to a same obsolescence rate that we have assumed for differential knowledge. Function g is a continuous and differentiable function indicating that the three information sets are relevant to produce new common information. The above discussion implies that the three arguments of the function should have, more or less, the same weight; an equal weight assumption is precisely the one taken to represent the following functional form:

$$g(\omega_t^{ad}, \omega_t^{bd}, \omega_t^c) = B(\omega_t^{ad}\omega_t^{bd}\omega_t^c)^{1/3}, \quad B > 0$$

Considering simultaneously difference equations (1), (2) and (3), and the initially presented definitions of ω_t^a and ω_t^b, we can rewrite the referred expressions under the form of a three-dimensional system where the endogenous variables are ω_t^a, ω_t^b and ω_t^c. The system under consideration is:

$$\begin{cases} \omega_{t+1}^a = (1-\delta)\omega_t^a + f(\omega_t^a)\xi^a(\omega_t^a, \omega_t^b) + g(\omega_t^a - \omega_t^c, \omega_t^b - \omega_t^c, \omega_t^c) \\ \omega_{t+1}^b = (1-\delta)\omega_t^b + f(\omega_t^b)\xi^b(\omega_t^a, \omega_t^b) + g(\omega_t^a - \omega_t^c, \omega_t^b - \omega_t^c, \omega_t^c) \\ \omega_{t+1}^c = (1-\delta)\omega_t^c + g(\omega_t^a - \omega_t^c, \omega_t^b - \omega_t^c, \omega_t^c) \end{cases}$$

The above system does not allow for a straightforward analysis of dynamic behavior; to undertake such study, one needs to resort to numerical examples. Consider the following array of parameters: $A = 0.5$, $B = 0.5$, $\delta = 0.25$, $\rho = 0.75$, $\eta = 2$.

So far, we have considered that the two firms engaged in the generation of information are completely similar. If they hold the exact same amount of information at the initial state, e.g., $\omega_0^a = \omega_0^b = 5$, then the outcome of their interaction will not differ from the case where only one media unit exists. For the assumed parameter values, this represents a stable outcome such that, in the long-run, $(\omega^a)^* = \left(\omega^b\right)^*$ corresponds to a fixed-point steady-state. However, this result changes if $\omega_0^a \neq \omega_0^b$ even when such difference is almost negligible; by considering $\omega_0^b = \omega_0^a + \varepsilon$, with ε an infinitesimal value (e.g., $\varepsilon = 10^{-15}$), the long-term outcome may deviate significantly from the fixed-point result with identical steady-states for both units. Figure 1 represents, for a small array of values of parameter A, a bifurcation diagram that shows how the system departs from the stability result; at $A = 0.5$, we find a result of chaos, i.e., the represented variable (information generated by media unit **A**) will display a completely irregular bounded instability behavior. Figure 2 confirms the chaotic outcome, by presenting an attractor that indicates all possible combinations of values of ω_t^a and ω_t^b in the long-run.

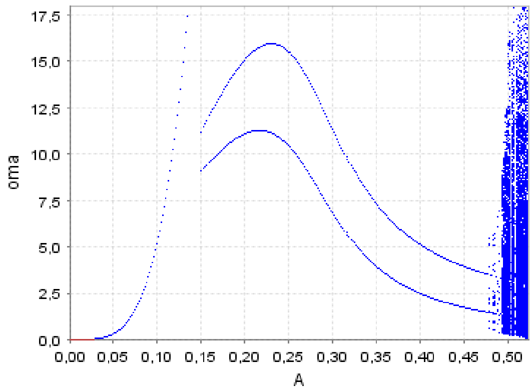

Figure 1. Bifurcation diagram (A, ω_0^a)

The relevant information broadcasted to the interested audience will be the sum of ω_t^{ad}, ω_t^{bd} and ω_t^c or, equivalently, $\omega_t^0 = \omega_t^a + \omega_t^b - \omega_t^c$. Figure 3 presents the time path of ω_t^0 after excluding the first one thousand observations. A cyclical pattern is found, indicating that the simultaneous competition and cooperation between media units, under the pattern of interaction we have considered, leads to periods of high quality information release and periods of low quality information release, that alternate in time.

Information is an economic good with peculiar properties. The above reasoning intends to capture some of these properties. First, although having features of a public good, namely a non-rival nature, it is not readily available in the economy for everyone to use—a purposeful effort in generating and processing information must be taken by firms that react to market incentives. Thus, it makes sense to consider the endogenous production of information in a competitive market.

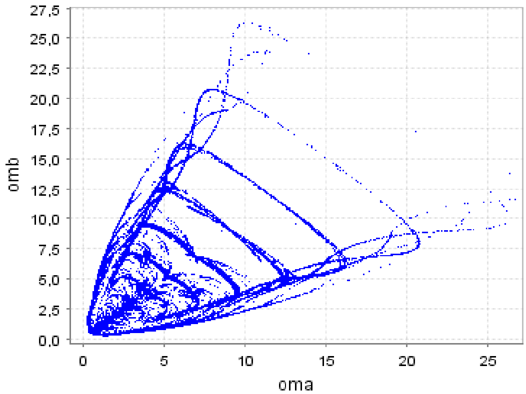

Figure 2. Chaotic attractor (ω_0^a, ω_0^b), $A = 0.5$

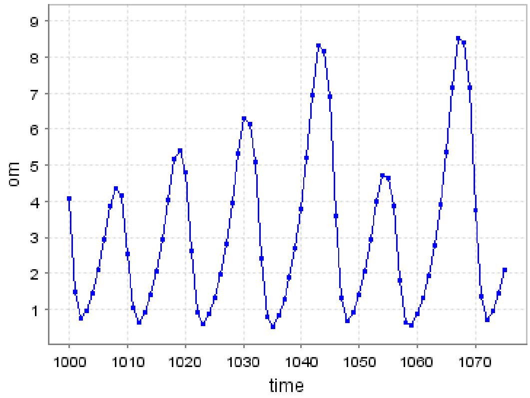

Figure 3. Time path of ω_t^0

It is also important to clarify the extent in which media firms are similar and in what they differ. We have referred that 'identical' media units interact; they are identical in one specific sense—they both act, within a given environment, in order to produce high quality information that they sell to the market. In other words, they have access to similar resources and technology and face identical constraints (regarding the impact of externalities and the obsolescence of information), and adopt a rational behavior given the available circumstances. This does not mean, though, that media units publish identical news. Heterogeneity is admissible at the level of the information they want to keep specific and at what grounds each unit intends to collaborate with the other. In synthesis, we might say that media firms are similar in terms of rational behavior in the market (they compete for the best information), but that they can be distinguished in what concerns the contents of the information they provide—nothing in our model indicates the absence of a plural disclosure of information.

3. A pattern of information diffusion

After having described how an irregularly cyclical flow of information might emerge in a context of competing and cooperative media companies, it is important to understand how this information spreads throughout the population or, more precisely, throughout the firms in the market that need to use such information in order to update their expectations. We assume that, at each time moment t, agents are aligned along a diffusion path relatively to an information source that sends a signal of intensity ω_t^0. The corresponding diffusion process, adapted from Young (2009), will be given by

$$n_{s+1} = n_s + \varphi \left[F(n_s) - n_s \right],$$

where $F(n_s)$ is the cumulative distribution function of n_s and $\varphi > 0$ is the diffusion rate. Variable n_s corresponds to the share of agents that are placed at position s relatively to the information source; $s = 0$ implies that agents acquire information with the exact same quality as it is produced, while a large integer s will mean that the interested agents will be located far from the information source. We consider that the signal loses value as it departs from the information source. Take parameters $\gamma, \upsilon \in (0,1)$; the signal depreciation rule is assumed to be:

$$\omega_{s+1} = (1-\gamma)\omega_s + \gamma \upsilon \omega_0, \quad \omega_0 = \omega_t^0$$

Finally, $\underline{\omega}$ will represent the required signal intensity/quality needed for the adoption of the available information. We can have cases in which the long-term value of the information, $\omega^* = \upsilon \omega_0$, is above $\underline{\omega}$ and other cases where it locates under the threshold level. If ω^* is above $\underline{\omega}$, then all agents will update information in this time period; if $\omega^* < \underline{\omega}$, then only a share of agents will update the information. This is shown in Figure 4; according to the figure, at a given time period t, one can determine the share of individual firms that is attentive to the arrival of new information: as we depart from the information source (larger s), the intensity of the signal weakens; if $\underline{\omega}$ is relatively low, there is a large percentage of agents that will acquire information at t (eventually, they all acquire it); a large $\underline{\omega}$ will lead to an outcome according to which only the agents closer to the source will pay attention to the information and update expectations accordingly.

In Figure 4, the upper panel shows the point of intersection of the information curve with boundary value $\underline{\omega}$; as displayed in the lower panel, that point of intersection will have correspondence on a specific share of information adopters that one can locate over the s-shaped diffusion curve.

Consider the same numerical example that in the previous section allowed for exemplifying the formation of information signals. Take also a normal distribution with average $\mu = 0.1$ and standard-deviation $\sigma = 0.1$, in order to characterize the cumulative distribution function of n_s; plus, take as well the following value for the diffusion

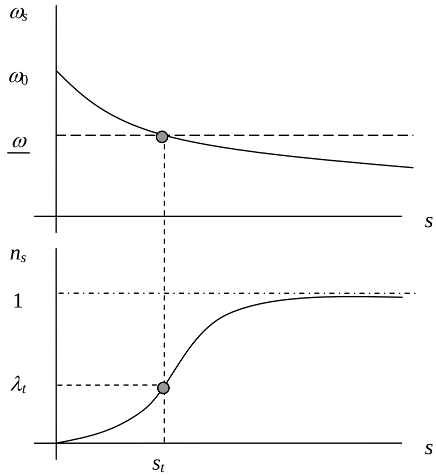

Figure 4. Process of information diffusion

rate $\varphi = 0.25$ and assume $\underline{\omega} = 1$, $\gamma = 0.2$ and $\upsilon = 0.25$. In the possession of the previous data, we are able to compute the value of the share of attentive agents at each time period; we will designate this share by the greek letter λ.

For some specific range of time periods, e.g., $t = 268, 269, \ldots, 276$, the application of the explained procedure leads to the following outcomes:

$$\begin{array}{lcl}
\omega_{268}^{0} = 7.4291 & \longrightarrow & \lambda_{268} = 1 \\
\omega_{269}^{0} = 5.0944 & \longrightarrow & \lambda_{269} = 1 \\
\omega_{270}^{0} = 2.0931 & \longrightarrow & \lambda_{270} = 0.5154 \\
\omega_{271}^{0} = 1.0902 & \longrightarrow & \lambda_{271} = 0 \\
\omega_{272}^{0} = 1.5049 & \longrightarrow & \lambda_{272} = 0.0980 \\
\omega_{273}^{0} = 2.2676 & \longrightarrow & \lambda_{273} = 0.6365 \\
\omega_{274}^{0} = 3.3053 & \longrightarrow & \lambda_{274} = 0.9137 \\
\omega_{275}^{0} = 4.679 & \longrightarrow & \lambda_{275} = 1 \\
\omega_{276}^{0} = 6.4519 & \longrightarrow & \lambda_{276} = 1
\end{array}$$

As the above list shows, there are different attentiveness rates for different time periods and the evolution of this rate follows an irregular pattern; this is the corollary of an information creation process that is subject to endogenous fluctuations.

4. The macroeconomic environment

Consider an economy that works under a monopolistically competitive market structure, producing many varieties of a single good. Each firm in the economy will produce a different variety. The central variables needed to characterize the proposed economic environment are the aggregate price level, which we represent in logarithmic form through variable p_t, and the output gap, that we represent by y_t and define as the difference, also in logarithms, between effective output and potential output. We also need to take a parameter $\alpha \in (0,1)$, which will represent the degree of real rigidities in the assumed economy or the degree of substitutability between the different varieties of the good this economy produces.

Following Mankiw and Reis (2002), every firm in this economy will solve the same profit maximization problem and arrive to the same expression for the desired price or target price:

$$p_t^* = p_t + \alpha y_t \tag{4}$$

Variable p_t^* is the target price; as stated, since all firms solve an identical problem, they will all want to charge an identical price. Parameter α plays a fundamental role in the selection of the optimal target price. When $\alpha = 0$, this implies that the varieties of the good are perfect substitutes; in such a competitive scenario, desired prices and observed price level coincide.[1] The larger the value of α, the less competitive the market is, and therefore the target price will depart from the price level; note, in particular, that in periods of recession ($y_t < 0$), we will have $p_t^* < p_t$, and the opposite in expansion phases.

In this setting in which all firms solve exactly the same maximization problem, there will be a source of heterogeneity attached to the degree of attentiveness evidenced by each productive unit. Firms will be inattentive to the arrival of new information and, thus, they will not necessarily update information at every time period.[2] As a result of this assumption, the selected price is not exactly the target price, as computed through the optimization problem, but instead the expectation about such desired price that was generated at the time moment corresponding to the date of the last updating of information. A firm that has updated its information about the state of the economy for the last time j periods ago will choose to set a price $p_t^j = E_{t-j}(p_t^*)$.[3]

To arrive to the aggregate price level, one needs to know how many firms, from the assumed universe, have updated their information, for the last time, at each period $t-j$. Following the notation of the previous section, let λ_{t-j} represent such share.[4] The aggregate price level can be presented as follows

$$p_t = \sum_{j=0}^{n} \lambda_{t-j} p_t^j, \tag{5}$$

[1] In the perfect competition scenario, $\alpha = 0$, the economy works under full efficiency and, thus, the output gap is also zero.

[2] As in Mankiw and Reis (2002), in our framework inattentiveness persists over time because firms face costs when collecting and processing information. The difference between the two models is that we explicitly consider a cause for this inattention to subsist: firms are differently located relatively to the information sources. Differences in dimension, geographical location, ability in managing information systems are reasons that explain why firms differ in the access to information.

[3] The expectation is conditional on the information the agent has about the state of the economy at time t, when formulating her forecast at time $t-j$.

[4] Recall that λ is formed through the interaction between the generation of information and the degree of attentiveness, as explained in Section 3.

where n corresponds to the distance in time relatively to the most recent time period in which 100% of the existing firms were simultaneously attentive to the arrival of new information.

At this stage, we can establish a bridge between the above simple framework used to characterize the behavior of inattentive firms in a monopolistically competitive market and the information creation and diffusion setting of the previous sections. We will use the numerical example that allowed, at the end of Section 3, to present specific shares of attentiveness. For the time periods $t = 268, 269, 275, 276$, there is no inattentiveness; all firms are aware of the contemporaneous relevant information to take decisions concerning the setting of prices and, as a result, there will be a coincidence between the observed price level and the target price; a corollary of this observation is that, at each one of these time periods, the output gap remains at its steady-state value $y_t = 0$.

For the other considered time periods, the reasoning is not so straightforward. Take $t = 270$; in this period, only 51.54% of the firms are attentive and resort to current information to set prices, while the remaining share, 48.46%, will use information from the preceding period to choose a price. As a result, and according to expression (5), the aggregate price level will be $p_t = 0.5154p_t^0 + 0.4846p_t^1$. Noticing that $p_t^0 = p_t^*$ and $p_t^1 = E_{t-1}(p_t^*)$, the aggregate price level expression can be rearranged and rewritten as

$$p_t = 1.0617\alpha y_t + E_{t-1}(p_t + \alpha y_t) \quad (t = 270).$$

The previous procedure is applicable to the other assumed periods. At $t = 271$, all firms in the market will be inattentive and, hence, they will all resort to outdated information; 51.54% of the firms will use information collected at $t-1$ (i.e., at $t = 270$), while the other 48.46% have updated information two periods before, at $t = 269$. Therefore, at this specific time moment, $p_t = 0.5154p_t^1 + 0.4846p_t^2$ or, resorting to the expectations' expressions,

$$p_t = 0.5154E_{t-1}(p_t + \alpha y_t) + 0.4846E_{t-2}(p_t + \alpha y_t) \quad (t = 271).$$

Proceeding to the following date, $t = 272$, observe that 9.8% of the population of firms updates information at this precise time period, while all the remaining will continue using information from the past. Recall that at $t-2$ ($t = 270$), 51.54% of the firms collected information; thus, the difference between these two shares (41.74%) will correspond to the production units who have updated their information for the last time two periods ago. The remaining value (48.46%) corresponds to the firms who have updated their information for the last time three periods ago, i.e., at $t = 269$. Thus, we can write the current aggregate price level as $p_t = 0.098p_t^0 + 0.4174p_t^2 + 0.4846p_t^3$, or, equivalently,

$$p_t = 0.1086\alpha y_t + 0.4628E_{t-2}(p_t + \alpha y_t) + 0.5372E_{t-3}(p_t + \alpha y_t) \quad (t = 272).$$

The remaining dates obey to a same logic. At $t = 273$, 63.65% of the firms collect and process new information; the remaining firms will be in the group of agents who have accessed information for the last time four periods before, at $t = 269$. Hence, the aggregate price level will correspond to $p_t = 0.6365p_t^0 + 0.3635p_t^4$, which is equivalent to

$$p_t = 1.751\alpha y_t + E_{t-4}(p_t + \alpha y_t) \quad (t = 273).$$

At period $t = 274$, there is a share of 91.37% of firms updating information, meaning that 8.63% of the firms have gathered information for the last time somewhere in the past; more precisely, they have collected relevant information at $t-5$ (i.e., $t = 269$). The price level is now $p_t = 0.9137p_t^0 + 0.0863p_t^5$, which is the same as

$$p_t = 10.5875\alpha y_t + E_{t-5}(p_t + \alpha y_t) \quad (t = 274).$$

By taking two simple assumptions that appear reasonable from an empirical point of view [(i) information does not flow smoothly; (ii) firms update information infrequently in time] we have found that the formation of the price level does not obey to a single and unchangeable in time rule. Instead, the formation of aggregate prices will be determined by specific conditions that are the result of how firms have behaved in a given range of past periods relatively to the information they could gather. As the next sections will turn clear, the impossibility of finding a unique price level expression for different time periods, will be decisive in terms of the characterization of macroeconomic behavior—the main consequence is the formation of an equilibrium result that is specific for the time period under consideration and that, most probably, will not be repeated in time. As stated in the introduction, this can be considered a way to approach complexity in macroeconomics: we start with an environment where there are slight forms of heterogeneity among agents, which generate outcomes that are not comparable from one period to the next; thus, there is not convergence (or divergence) relatively to any kind of long-term equilibrium, and the economy can only be characterized as remaining permanently outside a steady-state position. Each outcome, found at each period, is a unique result generated by a unique series of events and a unique interaction process, that may never repeat itself.

5. Sticky-information Phillips curves

In this section, we transform the price level expressions previously computed for each time moment into Phillips curves, i.e., into relations between the output gap and the inflation rate. The inflation rate is defined as the difference between the logarithms of prices at two consecutive periods: $\pi_t := p_t - p_{t-1}$.

To proceed, we need a definition of steady-state (although, as argued, in the previous section, the economy does not necessarily tend to converge to such state); in the steady-state, there will be full information, observed and desired prices will coincide and the output gap will be equal to zero, $y^* = 0$. Relatively to the steady-state inflation rate, π^*, we will be able to compute its precise value only after introducing the demand side of the economy later on.

We also need to specify a rule for the formation of expectations. This will follow the reasoning that we now characterize. Our expectations are of the type $E_{t-j}(p_t + \alpha y_t)$. If expectations are formed today relatively to some current event, then we know for certain what happens, i.e., there is perfect foresight, $E_t(p_t+\alpha y_t)=p_t+\alpha y_t$. If expectations are formed at $t-1$ relatively to an event occurring at t, perfect foresight may hold with some probability a that is eventually lower than 1; the alternative to perfect foresight is to think about moment t, at $t-1$, as the long run, and thus there will be a probability $1-a$ of having an expectation for an event at t that corresponds to the value at $t-1$ plus the steady-state growth rate required to place the value in the following period. Analytically, this means that, for a given probability a, we should consider the following expectations' rule: $E_{t-1}(p_t+\alpha y_t)=a(p_t+\alpha y_t)+(1-a)(p_{t-1}+\pi^*+\alpha y_{t-1})$. This rule can be generalized by taking the reasonable assumption that as we go back in time the probability of being possible to form expectations under perfect foresight progressively falls and, accordingly, the probability of considering the current period as the long-run steady-state rises. The corresponding rule, for any period j, will then be

$$E_{t-j}(p_t+\alpha y_t)=a^j(p_t+\alpha y_t)+(1-a^j)(p_{t-j}+j\pi^*+\alpha y_{t-j}). \tag{6}$$

The application of the rule of formation of expectations (6) to the series of price level expressions in section 4 will allow to find a series of Phillips curves. These curves are characterized by establishing a contemporaneous positive relation between the output gap and the inflation rate, but the inflation rate at each time period t will also depend on past values of inflation and output gap. Each Phillips curve has a peculiar shape as the result of the heterogeneity on the pattern of information diffusion we have characterized before. We apply the expectations' rule (6) to the computed price level equations (for $t=270,\dots,274$) in order to find five differently shaped Phillips curves. Computation allows to determine:

$$\begin{aligned}
\pi_t &= \frac{1.0617+a}{1-a}\alpha y_t+\alpha y_{t-1}+\pi^* \quad (t=270)\\
\pi_t &= \frac{0.5154a+0.4846a^2}{1-0.5154a-0.4846a^2}\alpha y_t-\frac{0.4846(1-a^2)}{1-0.5154a-0.4846a^2}\pi_{t-1}+\\
&\quad+\frac{0.5154(1-a)}{1-0.5154a-0.4846a^2}\alpha y_{t-1}+\frac{0.4846(1-a^2)}{1-0.5154a-0.4846a^2}\alpha y_{t-2}+\\
&\quad+\frac{1.4846-0.5154a-0.9692a^2}{1-0.5154a-0.4846a^2}\pi^* \quad (t=271)\\
\pi_t &= \frac{0.1086+0.4627a^2+0.5373a^3}{1-0.4627a^2-0.5373a^3}\alpha y_t-\pi_{t-1}-\frac{0.5373(1-a^3)}{1-0.4627a^2-0.5373a^3}\pi_{t-2}+\\
&\quad+\frac{0.4627(1-a^2)}{1-0.4627a^2-0.5373a^3}\alpha y_{t-2}+\frac{0.5373(1-a^3)}{1-0.4627a^2-0.5373a^3}\alpha y_{t-3}+\\
&\quad+\frac{2.5373-0.9254a^2-1.6119a^3}{1-0.4627a^2-0.5373a^3}\pi^* \quad (t=272)\\
\pi_t &= \frac{1.751+a^4}{1-a^4}\alpha y_t-\pi_{t-1}-\pi_{t-2}-\pi_{t-3}+\alpha y_{t-4}+4\pi^* \quad (t=273)\\
\pi_t &= \frac{10.5875+a^5}{1-a^5}\alpha y_t-\pi_{t-1}-\pi_{t-2}-\pi_{t-3}-\pi_{t-4}+\alpha y_{t-5}+5\pi^* \quad (t=274)
\end{aligned}$$

In order to turn the analysis of macroeconomic equilibria feasible, in the next section, we consider, henceforth, a specific value for probability a; let $a = 0.75$. This value implies that at $t-1$, there is a 75% probability of forming expectations through perfect foresight; this probability falls to 56.25% ($a^2 = 0.5625$) if the expectation is formed at $t-2$; it falls to 42.19% ($a^3 = 0.4219$) if the expectation is generated at $t-3$, and so forth. With these values, we simplify the sticky-information Phillips curves,

$$\begin{aligned}
\pi_t &= 7.2468\alpha y_t + \alpha y_{t-1} + \pi^* \quad (t = 270) \\
\pi_t &= 1.9337\alpha y_t - 0.622\pi_{t-1} + 0.378\alpha y_{t-1} + 0.622\alpha y_{t-2} + 1.622\pi^* \quad (t = 271) \\
\pi_t &= 1.1608\alpha y_t - \pi_{t-1} - 0.6053\pi_{t-2} + 0.3946\alpha y_{t-2} + \\
&\quad +0.6053\alpha y_{t-3} + 2.6053\pi^* \quad (t = 272) \\
\pi_t &= 3.0423\alpha y_t - \pi_{t-1} - \pi_{t-2} - \pi_{t-3} + \alpha y_{t-4} + 4\pi^* \quad (t = 273) \\
\pi_t &= 14.1928\alpha y_t - \pi_{t-1} - \pi_{t-2} - \pi_{t-3} - \pi_{t-4} + \alpha y_{t-5} + 5\pi^* \quad (t = 274)
\end{aligned}$$

The displayed equations remind us again of the complex nature of our setting; there is not a timeless rule governing the aggregate supply side of the described economy. The pattern of information diffusion introduces different parameter values at different time periods to describe the relation between the real state and evolution of the economy and the growth of the price level. In the next section, we will introduce the demand side and we will determine equilibrium levels for the endogenous variables. These equilibrium values will allow to perceive how the economy reacts to monetary policy shocks, given that economic conditions change systematically over time.

6. (Out-of-)equilibrium dynamics

In this section, we begin by introducing the demand side of the economy. Households solve a conventional intertemporal maximization problem; their objective function is an utility function that has as single argument the level of real consumption (labor market considerations are absent and, thus, leisure is not an argument of the objective function). In order to focus the analysis of inattentiveness on the supply side (i.e., on the behavior of firms), we assume that households are fully attentive when establishing their consumption plans.[5] Such setting allows to determine the following trivial rule for the evolution of the expected consumption level

$$E_t(c_{t+1}) = c_t + \theta r_t. \tag{7}$$

In equation (7), $\theta > 0$ is a parameter of the representative consumer's utility function, which represents the intertemporal elasticity of substitution between consumption

[5] Evidently, one could also establish a mechanism according to which consumers are differently located relatively to information sources and, thus, they would also display, as firms, different degrees of attentiveness, in this case concerning the elaboration of consumption plans. We have chosen to concentrate the analysis of attentiveness in the supply side, as Mankiw and Reis (2002), in order to allow for a better tractability of the model and to highlight how firm heterogeneity is enough to reshape at every instant the macroeconomic equilibria. Consumer inattentiveness would not add much in terms of the qualitative conclusions the model allows to reach.

in two consecutive time periods; variable c_t relates to the level of consumption and it is defined under the same terms as the output gap, i.e., it translates the difference between the logarithms of effective and potential consumption. Finally, r_t respects to the real interest rate; this rate is defined by the Fisher equation, $r_t = i_t - E_t(\pi_{t+1})$, where i_t corresponds to the nominal interest rate that is set by the central bank.

We consider that markets clear; this assumption, in our macroeconomic setting, just means that condition $y_t = c_t$ holds (consumption is the only component of demand in our simplified framework, where capital accumulation, public expenditures and external trade relations are absent). The path of the nominal interest rate is determined by the monetary authority, given the goals of monetary policy; these goals are twofold: price stability and real stabilization. A Taylor rule equation allows to synthesize how monetary policy is conducted; the rule will have the following shape:

$$i_t = \phi_y y_t + \phi_\pi \left[E_t(\pi_{t+1}) - \overline{\pi}\right] + \varepsilon_t \tag{8}$$

Taylor rule (8) involves two policy parameters, ϕ_y and ϕ_π. The first of these parameters reflects the concern with output stabilization; it will be any positive value (or zero, if the real stabilization concern is absent). Parameter ϕ_π reflects how the monetary authority reacts to changes on the expected inflation rate relatively to a given target value $\overline{\pi}$ that the central bank selects. Literature on this theme typically indicates that determinacy implies an active interest rate policy, under which the nominal interest rate must respond aggressively (by more than one to one) to changes on the expected inflation rate; in other words, the condition $\phi_\pi > 1$ must be imposed. Finally, variable ε_t is a disturbance term (it indicates other eventual determinants of the nominal interest rate, besides the ones mentioned in the analysis); we consider this to evolve under a white noise process. More specifically, we assume $\varepsilon_t \sim N(0, \sigma^2)$, with σ the standard deviation of the process.

Taking into account the market clearing condition, the Fisher equation and the Taylor rule, and assuming households are endowed with full information and perfect foresight, the consumption motion equation (7) is convertible on the expression

$$y_{t+1} = (1 + \theta\phi_y)y_t + \theta(\phi_\pi - 1)\pi_{t+1} - \theta\phi_\pi\overline{\pi} + \theta\varepsilon_t. \tag{9}$$

Equation (9) represents aggregate demand. Following the observation we have advanced before that the output gap is zero in the steady-state, we can resort to this equation to determine the steady-state level of the inflation rate, π^*, in the absence of interest rate volatility ($\varepsilon^* = 0$). The obtained result is $\pi^* = \overline{\pi}\phi_\pi/(\phi_\pi - 1)$. Observe that as long as $\phi_\pi > 1$, we have $\pi^* > \overline{\pi}$, i.e., the steady-state value of the inflation rate is larger than the target that the monetary authority establishes for the growth rate of the price level. This is an expected result since the Taylor rule is not an optimal monetary policy rule. We should also note that the more active the monetary policy is, the more π^* approaches $\overline{\pi}$. Taking into consideration π^*, we rewrite expression (9),

$$y_{t+1} = (1 + \theta\phi_y)y_t + \theta(\phi_\pi - 1)(\pi_{t+1} - \pi^*) + \theta\varepsilon_t. \tag{10}$$

The aggregate demand equation (10) should now be combined with the Phillips curves that were previously determined, in order to encounter pairs of equilibrium values (π_t, y_t). For time moments $t = 268, 269, 275, 276$, the output gap remains at its steady state value, $y_t = 0$, and thus it is straightforward to encounter the equilibrium level for the inflation rate

$$\pi_t = -\frac{1+\theta\phi_y}{\theta(\phi_\pi - 1)} y_{t-1} - \frac{1}{\phi_\pi - 1}\varepsilon_{t-1} + \pi^*.$$

Relatively to the other assumed time moments, one may find expressions for y_t by replacing the corresponding values in the Phillips curves into equation (10), the results are:

$$\begin{aligned}
t = 270: \quad y_t &= \frac{1}{1 - 7.2468\alpha\theta(\phi_\pi - 1)} \cdot \left\{[1 + \theta\phi_y + \alpha\theta(\phi_\pi - 1)]\, y_{t-1} + \theta\varepsilon_{t-1}\right\} \\
t = 271: \quad y_t &= \frac{1}{1 - 1.9337\alpha\theta(\phi_\pi - 1)} \cdot \{[1 + \theta\phi_y + 0.378\alpha\theta(\phi_\pi - 1)]\, y_{t-1} + \\
&\quad +0.622\alpha\theta(\phi_\pi - 1) y_{t-2} - 0.622\theta(\phi_\pi - 1)(\pi_{t-1} - \pi^*) + \theta\varepsilon_{t-1}\} \\
t = 272: \quad y_t &= \frac{1}{1 - 1.1608\alpha\theta(\phi_\pi - 1)} \cdot [(1 + \theta\phi_y) y_{t-1} + 0.3946\alpha\theta(\phi_\pi - 1) y_{t-2} \\
&\quad +0.6053\alpha\theta(\phi_\pi - 1) y_{t-3} - \theta(\phi_\pi - 1)(\pi_{t-1} - \pi^*) \\
&\quad -0.6053\theta(\phi_\pi - 1)(\pi_{t-2} - \pi^*) + \theta\varepsilon_{t-1}] \\
t = 273: \quad y_t &= \frac{1}{1 - 3.0423\alpha\theta(\phi_\pi - 1)} \cdot [(1 + \theta\phi_y) y_{t-1} + \alpha\theta(\phi_\pi - 1) y_{t-4} \\
&\quad -\theta(\phi_\pi - 1)(\pi_{t-1} - \pi^*) - \theta(\phi_\pi - 1)(\pi_{t-2} - \pi^*) \\
&\quad -\theta(\phi_\pi - 1)(\pi_{t-3} - \pi^*) + \theta\varepsilon_{t-1}] \\
t = 274: \quad y_t &= \frac{1}{1 - 14.1928\alpha\theta(\phi_\pi - 1)} \cdot [(1 + \theta\phi_y) y_{t-1} + \alpha\theta(\phi_\pi - 1) y_{t-5} \\
&\quad -\theta(\phi_\pi - 1)(\pi_{t-1} - \pi^*) - \theta(\phi_\pi - 1)(\pi_{t-2} - \pi^*) \\
&\quad -\theta(\phi_\pi - 1)(\pi_{t-3} - \pi^*) - \theta(\phi_\pi - 1)(\pi_{t-4} - \pi^*) + \theta\varepsilon_{t-1}]
\end{aligned}$$

The above expressions give us the contemporaneous values of the output gap as functions of the output gap and inflation rate in previous periods; note that the disturbance term that is present in the Taylor rule has a role here: shocks occurring at $t-1$ exert an effect over the output gap in t. To obtain inflation rates at t depending solely on past values of the endogenous variables, one just needs to replace the above expressions for y_t into the Phillips curves of the previous section.

If one assumes $\varepsilon_t = 0\ \forall t$, the equilibrium values (π_t, y_t) will remain at the steady-state level $(\pi^*, 0)$. Eventual disturbances will imply a departure from the steady-state, and this departure will be of a different nature when occurring at different time periods, since we have seen that each time period is characterized by a different supply side relation. The next paragraphs exemplify how this economy will work in the time span one has considered.

Take the following parameter values: $\theta = 1$ (logarithmic utility), $\alpha = 1$ (the degree of real rigidities is relatively low), $\overline{\pi} = 0.01$ (the central bank chooses an inflation target of 1%), and $\phi_y = 0.5$, $\phi_\pi = 1.5$ (these are standard values in monetary policy analysis). Consider, as well, that the disturbance term ε_t is given by a normal distribution with zero mean and standard deviation equal to 0.001. Under this setting, the steady-state level of the inflation rate is $\pi^* = 0.03$, three times the value of the inflation target selected by the monetary authority.

With the assumed parameter values, we compute the paths of the output gap and of the inflation rate for the time interval we have considered ($t = 268$ to $t = 276$). Values of ε_t will be different each time we run the model. Figure 5 presents three panels, each one for one possible trajectory of the disturbance variable. As one can observe, these are completely uncorrelated—no common trace is identifiable between them. Next, we apply each one of these disturbance paths to the macroeconomic relations one has built. Now, we encounter some common features, as illustrated in Figure 6: the output gap remains equal to zero in the two first time periods and in the last two time periods, the moments in which full information prevails, and it is clear that deviations relatively to the steady-state become more striking as we accumulate periods of departure from full information; as the full information scenario is recovered at periods $t = 275$ and $t = 276$, the economy returns to the steady-state.

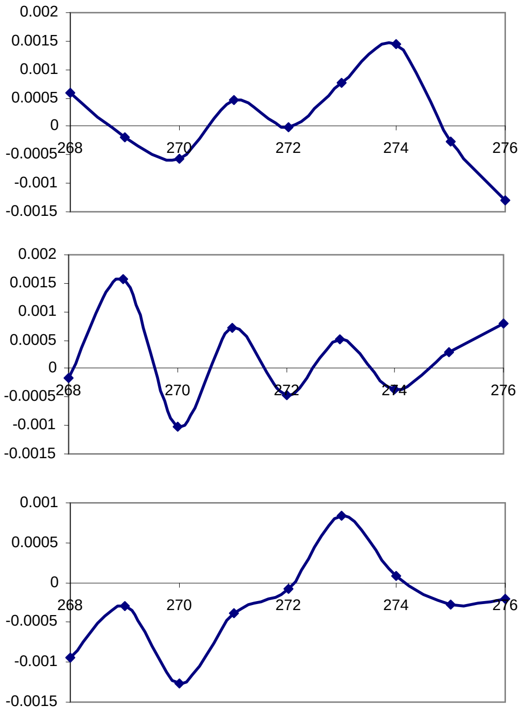

Figure 5. Possible time trajectories of the white noise value ε_t

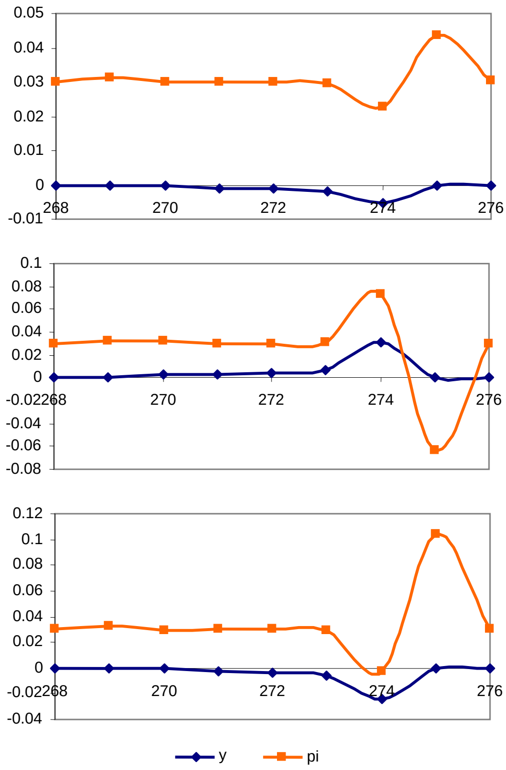

Figure 6. Possible time trajectories of variables π_t and y_t

The main lesson to withdraw from the analysis is that current macroeconomic results are necessarily the result of how the economy has worked in the past. Because of barriers encountered by firms concerning the possibility of collecting and processing useful information, their behavior will depart from the benchmark full information case, with the departures relatively to the steady-state being enlarged each time the economy is incapable of providing to all firms the information that is necessary to set the desired prices. Thus, assuming that complete information at every time moment is unfeasible, economic outcomes are generally deviated from equilibrium (with equilibrium interpreted as the steady-state), historically determined and the result of complex patterns of interaction between agents that behave optimally but are eventually subject to small differences in initial endowments (in the case, media may have slightly different initial endowments of information resources).

The results also point to a mechanism of self-adjustment that the economy eventually follows. The economy may depart significantly from equilibrium, what implies a stronger coverage of economic news by the media, allowing firms to be more attentive and to revise their plans on a more frequent basis; this may lead the economic system to return to the steady-state position. Once in the steady-state, the news on the performance of economic aggregates lose interest, less quality information is produced and the degree of attentiveness by firms falls; then, a new phase of departure relatively to the equilibrium values is initiated, and this process may repeat itself endlessly. In this way, a cyclical pattern on information release is generated, implying simultaneously a cyclical pattern on the evolution of real and nominal economic aggregates.

7. Conclusion

In the introduction, we have referred six characteristics of complex systems (agents' heterogeneity, interaction, evolution, path-dependency, self-organization, out-of-equilibrium dynamics). The macroeconomy is necessarily, in the light of such characteristics, a complex environment and should be approached as such. The main question is how much one needs to deviate from mainstream economic analysis in order to include the mentioned properties in a meaningful characterization of the macroeconomic system. The answer can be just a few minor changes over an otherwise benchmark aggregate model.

We have resorted to the Mankiw-Reis sticky-information setting and adapted it in order to include some of the main features of a complex system. We began by assuming an endogenous process of generation and diffusion of information; at this level, a small difference on the initial endowment of informational resources by competing media companies may trigger a process of irregular information flows that culminates in a cyclical pattern of information acquisition by price-setting firms: in some time periods, all firms access relevant information (because this is readily available), while in other periods information is scarce and only a few agents (or even none) have access to it. Thus, the first of the properties of a complex system is inherent to the analysis: endogenous fluctuations arise only if some kind of heterogeneity (even if almost negligible) is considered. In our framework, two sources of heterogeneity are present: small differences between media units and different positions of the price-setting firms relatively to the information source.

Once heterogeneity is explicitly modelled, the other properties arrive naturally: media units interact, by competing and cooperating, and this allows for the erratic trajectory observed for the production of information. Evolution and adaptation are present in the behavior of the media companies and also in the behavior of firms, that change the way they form expectations given the information they possess. As a result, each time period will be characterized by a different supply-side relation, what indicates that macro relations are history dependent (there is path-dependency) in a context where the interaction creates a scenario of never ending evolution that is self-organized. As figure 6 shows, a small disturbance may provoke relevant departures from equilibrium, and the economy may evolve following some bounded instability path.

Models are simplifications of the reality and they need to accomplish a compromise between simplicity and comprehensiveness. Macroeconomic models, in particular, should be simple representations of aggregate phenomena, but aggregate phenomena, by definition, are not simple. They are the outcome of millions of individual decisions that are taken, at a daily basis, by many thousands of households and firms who have to make choices in order to attain their pre-specified goals. The field of macroeconomics has built some important tools to better understand issues as long-run growth, business cycles, economic policy, among others. However, all of this was constructed under the premise that one can explain the functioning of the whole of the economy by analyzing the behavior of a representative or median agent and by studying time invariant aggregate relations. Although macro models are extremely useful to achieve a good perception on the functioning of the economic system, some steps have to be taken in the direction of increased realism. By including behavior heterogeneity in the analysis of macroeconomic relations, one introduces new important features for the understanding of such relations. Persistent disequilibrium and path dependence, which are empirically observed phenomena, originate on this kind of setting.

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References

Arthur, W. B., Durlauf, S. N. and Lane, D. A. (1997). Introduction. In Arthur, W. B., Durlauf, S. N. and Lane, D. A., *The Economy as an Evolving Complex System II*. Addison-Wesley, Redwood City, 1–14.

Colander, D. (2008). Complexity and the History of Economic Though. Middlebury, Middlebury College, Department of Economics, Working Paper No. 0804.

Fontana, M. (2008). The Complexity Approach to Economics: A Paradigm Shift. Turin, University of Turin, CESMEP Working Paper No. 200801.

Fontana, M. (2010). Can Neoclassical Economics Handle Complexity? The Fallacy of the Oil Spot Dynamic. *Journal of Economic Behavior & Organization*, 76, 584–596.

Fujita, M. (2009). Dynamics of Innovation Fields with Endogenous Heterogeneity of People. In Karlsson, C., Andersson, A. E. and Cheshire, P. C. (eds.), *New Directions in Regional Economic Development*. Berlin-Heidelberg, Springer-Verlag, 59–78.

Gomes, O. (2008). Too Much of a Good Thing: Endogenous Business Cycles Generated by Bounded Technological Progress. *Economic Modelling*, 25, 933–945.

Holt, R. P. F., Rosser, J. B. and Colander, D. (2010). The Complexity Era in Economics. Middlebury, Middlebury College, Department of Economics, Working Paper No. 1001.

Mankiw, N. G. and Reis, R. (2002). Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *Quarterly Journal of Economics*, 117, 1295–1328.

Mankiw, N. G. and Reis, R. (2003). Sticky Information: A Model of Monetary Nonneutrality and Structural Slumps. In Aghion, P., Frydman, R., Stiglitz, J. E. and Woodford, M. (eds.), *Knowledge, Information and Expectations in Modern Macroeconomics: In honor of Edmund S. Phelps*. Princeton, Princeton University Press, 64–86.

Mankiw, N. G. and Reis, R. (2006). Pervasive Stickiness. *American Economic Review*, 96, 164–169.

Mankiw, N. G. and Reis, R. (2007). Sticky Information in General Equilibrium. *Journal of the European Economic Association*, 5, 603–613.

Mantegna, R. and Stanley, H. E. (2000). *Introduction to Econophysics*. New York, Cambridge University Press.

Markose, S. M. (2005). Computability and Evolutionary Complexity: Markets as Complex Adaptive Systems (CAS). *Economic Journal*, 115, 159–192.

Martin, R. and Sunley, P. (2007). Complexity Thinking and Evolutionary Economic Geography. *Journal of Economic Geography*, 7, 573–601.

Puu, T. (2010). On the Economics of Increasing Complexity: with Some Special Focus on Culture. *Journal of Economic Behavior & Organization*, 75, 59–68.

Reis, R. (2009). A Sticky-Information General-Equilibrium Model for Policy Analysis. In Schmidt-Hebbel, K., Walsh, K. and Loayza, N. (eds.), *Monetary Policy under Uncertainty and Learning. Central Banking, Analysis and Economic Policies Book Series, Vol. 13*. Santiago de Chile, Central Bank of Chile, 227–283.!!!!!

Rosser, J. B. (1999). On the Complexities of Complex Economic Dynamics. *Journal of Economic Perspectives*, 13, 169–192.

Rosser, J. B. (2008). Econophysics And Economic Complexity. *Advances in Complex Systems*, 11, 745–760.

Velupillai, K. V. (2005). A Primer on the Tools and Concepts of Computable Economics. In Velupillai, K. V. (ed.), *Computability, Complexity and Constructivity in Economic Analysis*. Victoria: Blackwell, 148–197.

Young, H. P. (2009). Innovation Diffusion in Heterogeneous Populations: Contagion, Social Influence, and Social Learning. *American Economic Review*, 99, 1899–1924.